

Applied/Computational Mathematics

Guidelines for the MS and Ph.D in Applied and Computational Mathematics (Rev. 0.3, January 2005)

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1 The MS and PhD Programs in Computational Mathematics

This document addresses the details required to implement advanced degrees in applied and computational mathematics which are offered through the Department of Mathematics. These courses of study are specifically aimed at students seeking advanced graduate degrees in applied and computational mathematics. The MS degree option in applied and computation mathematics is an extension of the traditional mathematics MS degree offered by the Department, while the PhD degree offered through the Department of Mathematics is formally designated as a doctoral program in Computational Science with an emphasis in applied and computational mathematics (ACM).

2 Purpose

The graduate curriculum in mathematics has been restructured to service students who are interested in attaining advanced degrees in mathematics, and is designed to provide students with a solid background in applied and computational mathematics: The masters (MS) degree is structured so as to develop a student's background in applied mathematics and analysis while the doctoral degree (Ph.D) offered through the Department of Mathematics prepares students more intensively for careers in computational mathematics. Both degree offerings break new ground, and represents a change from the the traditional offerings at the graduate level at USM even at the MS level. The MS curriculum has been restructured so as to keep many of the existing courses, at most modifying the syllabi to be more consistent with the needs of students in applied and computational mathematics. The doctoral program rests on these course, but also builds on these by providing new doctoral level courses suitable for students in computational mathematics. The new ACM curriculum contains these courses at the MS level in MAT 560 *Numerical Analysis I*, MAT 606 *Partial Differential Equations* , and MAT 610 *Numerical Linear Algebra*, emphasizing more strongly the mathematical development of the material, and in MAT 7xx at the doctoral level which provides a more balanced approach between mathematical analysis and algorithmic issues. The changes at the MS level in course content is driven by the need to have the MS curriculum serve as a feeder for the PhD program, and to have a base of appropriate mathematical courses for doctoral students in applied mathematics.

The new, revised ACM curriculum is intended to provide an intense, focused study program consisting of courses which emphasize developing foundational skills in applied mathematics, complementing these at the doctoral level with courses that provide exposure to advanced topics in numerical methods in computational mathematical science. The course offering is also designed to provide a base curriculum in applied and computational mathematics which can be used by other departments within the College of Science Technology as service courses for their upper divisional, or graduate students, and as such the need to introduce economies of scale in teaching has been recognized and every attempt has been made to address these needs in developing and integrating this program into the overall computational science programs offered through various departments in the College of Science and Technology (CoST). In particular, the revised graduate courses attempt to integrate the curriculum with the courses offered in the newly devolved Computational Science Programs in Physics and in Computer Science and Statistics.

While the classes which are offered at the MS level are closely allied to the courses that were offered in the traditional MS program, the required coursework does not include more traditional topics in combinatorics, number theory, topology, and abstract algebra. Qualified students, however, are encouraged to take participate in this broader curriculum, and the particular electives which a student choose are determined by the student and their graduate committee. Primarily, the curriculum is designed to provide intense preparation in the area of applied mathematics, numerical analysis, and scientific computing.

3 Student admission, progress and degree requirements

Students participating in the new program can enter at the MS or PhD levels. Students with an appropriate BS degree only can choose to be admitted into the MS program, or directly into the PhD program, however in the latter case, these students will be required to complete the Computational Mathematics MS degree with the thesis option (a research MS) before being allowed to move on to doctoral student status; in this case the thesis will serve as the foundational document for the Prospectus for the doctoral research and students will not submit to the formal degree at the masters level if they are successful at being admitted to doctoral candidacy status, and thus are not required to take the MS Comprehensive Exams. Students with an appropriate MS degree may choose to apply for admission to the Computational Mathematics MS Program to do a further Masters in Applied and Computational Mathematics only if their previous degrees are not in this field. Students with an appropriate MS and BS degrees (e.g., in mathematics, engineering, etc.) may apply to the Computational Mathematics Program.

Students intent on participating at the MS level are encouraged to write an MS thesis (which will entail enrolling in two additional 3-credit research courses). The MS thesis and the associated research work forms the basis for continuing research by those students wishing to continue in their studies by enrolling as candidates for the PhD in Applied and Computational Mathematics. Writing a thesis in the Department of Mathematics has been an option rarely exercised, however for the new Computation Mathematics MS degree, the project provides the necessary validation that the student

can successfully integrate theory with application in a problem-solving environment. Thus both MS and PhD candidates will be required to prepare at a minimum a 20 to 50 page research paper during their course of study; For the student pursuing only a Masters degree, this research paper qualifies as the thesis, while for the doctoral candidate this serves as the formal document for the Prospectus presentation. Both MS and PhD students must present and defend their results before their graduate committees in order to successfully complete their MS program of study.

3.1 Admittance and progression through the Program: General degree requirements

In order to successfully attain the degree MS degree, the student admitted to this course of study must: 1) fulfill the coursework requirements laid down by the Department in regard to the minimum number of credit hours which must be completed and must also satisfy the requirements for satisfactory performance in all of the courses in which they have enrolled; 2) Must pass a comprehensive exam covering the core subjects; and either 3) must successfully defend a masters thesis if they have elected to pursue the Computational MS degree with a thesis option; or, 4) take an additional comprehensive on the material covered in the electives taken if following a non-thesis option.

In order to successfully attain the PhD degree, the student admitted to this course of study must: 1) fulfill the coursework requirements laid down by the Department in regard to the minimum number of credit hours which must be completed and must also satisfy the requirements for satisfactory performance in all of the courses in which they have enrolled; 2) Must pass a comprehensive exam covering the core subjects; 3) must successfully defend their Prospectus for Research; and 4) must complete and successfully defend their Dissertation which supports their research. The comprehensive exam will cover the core subjects areas associated with the Mathematics MS degree, and the core study areas specified in the PhD program of study.

BS qualified students who enrolled in the PhD program and who successfully complete their coursework, comprehensive exams, and their prospectus defense, but elect to leave the program and not go on for the doctorate, may, at the discretion of their graduate committees, formally choose to submit the results of their research as an MS thesis, and upon fulfilling the requirements of the graduate school, will be awarded an MS degree.

As shown in Fig. 1 there are several options available for students at all levels in the computational mathematics program.

3.2 University MS and PhD degree Requirements

The MS curriculum consist of 36 credit hours, which includes an option for graduate research through an MS thesis. Students are required to enroll in nine 3-credit core courses as listed in Table 1 and outlined in greater detail in Section 5.3. Students who are not pursuing the thesis option are also required to take two 3 hour topics courses, and these can be taken from any appropriate area in mathematics, applied science (e.g., physics, bio-informatics, etc.), or computer science subject to review by the students graduate committee. Possible electives which can fulfill this option within the Department of Mathematics are listed in Table 2. As a rule courses at or below the 400 level

cannot count toward graduate credit. In addition, MS students are required to complete at least 3 hours of a tools class. While MS students do not have an explicit requirement for seminar, students, particularly those interested in the MS by research, are strongly encouraged to participate in the Departmental Colloquia and the College Computational Science Seminar series.

The total hours required by the student to complete the MS degree is $9 \times 3 + 3 + 6 = 36$ credit hours, accounting for the nine core courses and two research courses if the student is following the thesis option, or nine core course and two elective courses if not. Students support on teaching assistantships will be expected to teach or perform other duties for the Department amounting to 20 hour per week during each Fall and Spring semester. Typically this will mean a load of 2 to 3 graduate courses per semester.

Students following the course of study leading to a PhD degree are required to take a total of at least 18 credit hours past the MS degree (i.e., a total of 54 credit hours beyond the BS degree of which 36 are in common with the MS curriculum). The doctoral curriculum consists of three 3-credit hour core courses at the 700 level, along with three 3-credit hour tools classes, and two 1-credit hour seminar classes, for a total of $3 \times 3 + 3 \times 3 + 2 = 20$ credit hours.

3.3 College PhD degree Requirements

The three 3-credit hour courses on research methods, or tools classes, are offered through the College of Science and Technology by the participating computational science departments (e.g., Mathematics or Computer Science), and are designed to assist the student in working in the area of computational science. Topics covered include utilizing computational resources, and using standard tools, software libraries, and techniques for writing scientific publications. It is also recommended that students with backgrounds deficient in computer science use elective courses to augment their skills in programming, and in all cases students entering the doctoral program will be required to be able to write basic code in an object oriented language such as C++, or Java, and may also be required to code in C and Fortran (as pertains to their research). While this is not a requirement for MS students, those interested in continuing on to doctoral research should bear in mind that competence in algorithmic and computational methods is an important consideration in admittance to the doctoral program.

3.4 Departmental Graduate Degree Requirements

The coursework with the requisite class exams is structured so as to provide only a midterm and final, or only a final for each course so as to avoid losing class contact hours to exams. Both MS by research and PhD candidates are required to pass a comprehensive exam upon completing their respective core curriculum. Students who are candidates for the PhD who successfully pass the comprehensive exams will be admitted to PhD status, and will be required to first successfully present and defend their prospectus for research, and then to defend their dissertations in order to complete the doctorate. Student who are following the MS curriculum are required to successfully present their thesis in order to obtain their degrees. The purpose of the comprehensive exam is to assess

Core Graduate Courses (27 hrs)	Tools Courses (3 hrs)	Electives (6 hrs)	Thesis (6 hrs)
MAT 526 Linear Algebra II MAT 541 Advanced Calculus I MAT 542 Advanced Calculus II MAT 560 Numerical Analysis II MAT 605 ODEs MAT 606 PDEs MAT 610 Num. Linear Algebra I MAT 636 Complex Analysis I MAT 641 Real Analysis I	CS 701, 702, and 703 offered through CoST	Details listed in Table 2 for courses offered by the Department of Mathematics	MAT 689

Table 1: MS Computational Mathematics courses.

mastery of theory by the student. The comprehensive exam for both MS courses of study will cover the material in the core curriculum. For MS students not taking the thesis option, the comprehensive exam will also include material from the elective courses chosen.

The MS thesis option requires two 3-credit research courses or 6 credit hours during which time the student's work will be supervised by a faculty advisor on their project. The MS thesis must be completed before the end of the term in which the student intends to graduate, hence students will be strongly encouraged to develop ideas, and to approach faculty in regard to their research interests early in their studies. As noted, this MS thesis can serve as the basis for the dissertation research conducted by doctoral students, and thus typically serves at the basis for the students prospectus presentation. Ideally, the student should already have begun background reading in research areas which are of interest late in the first term.

3.5 Student Committees

The progress of all incoming MS and PhD students is monitored by a faculty advisor. All incoming MS and PhD students will have the same faculty advisor initially, however at the time that a student begins to prepare for their research (either at the MS level when following the MS by research degree) or at the PhD level prior to preparing for the prospectus presentation, the student will form a graduate committee, and the chair of that committee assumes responsibility for monitoring the academic progress of the student, and in particular assumes the responsibility of monitoring and supervising the student's research. In any case, a student must first pass the comprehensive exams in order to be allowed to move on to research status and thus to officially form their graduate committees.

The research for the PhD constitutes the essence of the doctoral degree. This is first and foremost a degree which requires that the student not only master the state of the art in their chosen area of

specialization in computational mathematics, but also extend it. Students who enter the doctoral program, either as MS students or as BS students who are nearing completion of their course of study through the MS, are encouraged to seek out a faculty research advisor who is willing and able to work with them on a research topic, and who will become the chair of their doctoral committee when this is formed.

4 The MS courses

Students are required at a minimum to complete nine 3-credit core courses, one tools class, and either 2 elective classes or 2 research classes.

4.1 Syllabus for the Core MS Computational Math Curriculum

The proposed syllabus consists of foundational courses in linear algebra and analysis and differential equations, along with a range of introductory courses to topics in numerical analysis which complement the theory. The core is designed to cover fundamental topics in applied and computational mathematics, with a focus primarily aimed at providing MS students a solid, classical background in the numerical solution of differential and partial differential equations.

Each course is 3 credits, unless otherwise noted and each course contains a brief course description which attempts to highlight the major topics which will be covered, showing at the end of each description possible connections with other courses in the curriculum and any prerequisites which may be required. The course descriptions are not meant to be definitive.

1. **Linear Algebra II, MAT 526.** *An introduction to the structure and properties of linear transformations and their matrix representation.*

- Review of linear transformations, eigenvalues and eigenvectors, coordinate transformations and diagonalizability.
- Gram-Schmidt orthogonalization, direct sums, symmetric operators, bilinear forms and quadratic forms.
- The structure of linear operators, the Cayley-Hamilton Theorem, the development of Jordan canonical forms.
- Special matrices, e.g., positive definite, stochastic, Toeplitz, etc.

<i>Curriculum Objectives and Requirements</i>	The course is designed to develop the framework for the analysis which is required in numerical linear algebra, MAT 610. The course requires a fundamental background in elementary linear algebra, i.e., MAT 326 or MAT 426.
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2. **Advanced Calculus I, MAT 541.** *Foundational topics in analysis in \mathbb{R}^1 and \mathbb{R}^n .*

- Fundamentals of point set topology. Open and closed sets. Continuity, compactness.

- Norms, metric spaces, inner products, and fundamental results related to working with analytical estimates.
- Standard inequalities in analysis.
- Infinite series. Concepts in convergence of sequences and series of functions. Uniform convergence and approximation.

<i>Curriculum Objectives and Requirements</i>	Provides an elementary introduction to the basic ideas in mathematical analysis. Students who have already had a course in Advanced Calculus are exempt from this core requirement and may substitute an elective.
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3. **Advanced Calculus II, MAT 542.** *Vector calculus and vector valued functions in \mathbb{R}^3 .*

- Multivariable calculus: vector valued functions, vector analysis.
- The derivative and integral in \mathbb{R}^3 .
- Vector valued functions, the Stokes Theorems.

<i>Curriculum Objectives and Requirements</i>	Provides an elementary introduction to the basic ideas in mathematical analysis. Students who have already had a course in Advanced Calculus are exempt from this core requirement.
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4. **Numerical Analysis, MAT 560.** *Foundational concepts associated with the discretization of functions and the elementary numerical techniques used to work with these approximations.*

- Numerical error, and quantifying errors due to discretization and approximation. Norms.
- Methods for approximation and interpolating data, and methods for approximating functions.
- Numerical differential including an introduction to finite difference methods, and integration using Newton-Cotes and Gaussian quadrature.
- Finding the roots of equations. Nonlinear iteration.
- Adaptive methods.

<i>Curriculum Objectives and Requirements</i>	This course provides a foundational introductory background to numerical methods. Through the use of software packages, student develop a connection between theory and practice, and develop and understanding for the effectiveness of the numerical methods. The discrete representation of functions is developed, and fundamental numerical methods for working with discrete representations are developed. Requirements are a good working knowledge of calculus and introductory linear algebra.
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5. **Differential Equations, MAT 605.** *The course develops techniques for solving linear problems in differential equations in a formal framework encompassed by linear analysis.*

- Linear Transformation, and linear differential equations. Equations with constant coefficients, the Laplace transform, first order equations, the Wronskian, non-homogeneous equations, variation of parameter, reduction in order, method of undetermined coefficients, and homogeneous equations.
- Initial value and boundary value problems for ODEs: stiff equations; concepts of stability and convergence.
- Finite difference equations.

<i>Curriculum Objectives and Requirements</i>	The theory of differential equations is quite broad. In this, fundamental concepts from the classical theory of ODEs is developed and broadened to include numerical methods for solving these problems.
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6. **Partial Differential Equations, MAT 606.** *The classical theory of partial differential equations (PDEs) in two variable in \mathbb{R}^2 is developed and extended to developing numerical solution methods for elliptic, parabolic, and hyperbolic PDEs.*

- Theory of PDEs: scalar first order partial differential equations; characteristics, weak solutions. The wave equation in higher dimensions.
- Classification of linear second-order partial differential equations for hyperbolic elliptic and parabolic equations. Classical approaches to solving PDEs in \mathbb{R}^3 : Separation of variables, Green's functions.
- Solutions of parabolic/elliptic equations by finite differences; development of explicit and implicit methods in one dimension; The Lax Equivalence Theorem: error, stability and convergence; truncation error; Fourier and Von Neumann stability; maximum principles; boundary conditions; two and three dimensional problems. ADI methods.
- Finite differences for hyperbolic problems: CFL condition, Fourier analysis of stability, amplitude and phase errors; Lax-Wendroff for conservation laws, box and leap-frog methods; boundary conditions and conservation properties.
- Linear elliptic equations in two dimensions: difference schemes, finite volume approach; curved boundaries; error analysis using maximum principles.

<i>Curriculum Objectives and Requirements</i>	Partial differential equations form the core of most problems encountered in applied and computational mathematics; This course is aimed at integrating theory with practice in developing classical numerical methods for solving PDES. This course is aimed at integrating theory with practice in developing classical numerical methods for solving PDES. The course requires MAT 460 or MAT 560, or equivalent.
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7. **Numerical Linear Algebra, MAT 610.** *An introduction to basis tools and techniques in numerical linear algebra*

- Common problems in linear algebra. Matrix structure and developing methods for efficiently computing with matrix structures.
- Direct solution methods for linear systems, Gaussian elimination and its variants, including LU decomposition. Conditioning, and other numerical difficulties in solving linear systems.
- Iterative solution methods for linear systems including relaxation methods, and the conjugate gradient method. Convergence and rates of convergence of iterative methods.
- QR factorization, the QR algorithm for eigenvalues.

<i>Curriculum Objectives and Requirements</i>	This course is intended to cover a broad range of material which includes direct and iterative methods, and emphasizing the need to assess computational and algorithmic efficiency. Requires a foundational course in linear algebra, e.g., MAT 326 or MAT 426.
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8. **Complex Analysis I, MAT 636** *Foundational topics in complex analysis as used in applied mathematics.*

- Functions of a complex variable. Differentiating and integrating functions of, branches, contour integration.
- Complex analysis, complex potential and conformal mapping, complex integrals.
- Series. Special functions, orthogonal polynomials.

<i>Curriculum Objectives and Requirements</i>	The course provides the bridge between the theory of functions of a complex variable, and the numerous examples in which this theory is used in developing
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9. **Real Analysis I, MAT 641.** *Foundational topics in analysis.*

- The Topology of Cartesian Spaces. Continuity and limits. Compactness, Bolzano-Weierstrass property.
- Functions of one variable: Convergence, continuity, properties of the Riemann-Stieltjes Integral, the Riesz Representation Theorem,
- Sequences and series of functions, special functions. Uniform convergence, differentiation and integration of series, including Fourier series.
- Functions of several variables. Derivatives and integrals. Linear operators, Linear functionals.

<i>Curriculum Objectives and Requirements</i>	This course is intended to provide a rigorous introduction to fundamental ideas in real analysis. Advanced Calculus, MAT 541 and 542, are prerequisites.
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Topology/Geometry MAT 572 Modern Geometry MAT 575 General Topology MAT 601 Differential Geometry MAT 683 Topics in Topology/Geometry	Optimization & Numerical Analysis AT 518 Linear Programming MAT 519 Optimization in Math Programming MAT 685 Topics in Computational Math
Analysis MAT 636 Functions of a Complex Variable II MAT 682 Topics in Analysis MAT 642 Real Analysis II	Combinatorics & Graph Theory MAT 537 Graph Theory MAT 539 Combinatorics MAT 629 Applied Combinatorics & Graph Theory
Algebra & Linear Algebra MAT 521 Number Theory MAT 523 Modern Algebra I MAT 526 Linear Algebra II MAT 603 Modern Algebra MAT 681 Topics in Algebra	Applied Analysis MAT 520 Probability & Statistics II MAT 684 Topics in Applied Math MAT 685 Topics in Computational Math

Table 2: Elective graduate courses offered through the Department of Mathematics.

4.2 Elective Courses for the MS degree

The Department of Mathematics elective courses are listed in Table 2, and any course with the MAT 5xx/6xx or CS 7xx approved by the student's graduate committee is suitable for satisfying any elective requirements,

Elective courses typically can be chosen by the student to be any area of mathematics (each year an appropriate topics course will be offered in the Department of Mathematics), or in any related field of study, offered in the College of Science and Technology. Students interested in pursuing the PhD in Computational Science in Mathematics who do not have a strong background in computing are encouraged to enroll in additional elective courses in the Department of Computer Science.

Students who are interested in working in an interdisciplinary setting on a computational science graduate degree, for example in computational fluid dynamics, or computational biology may benefit from additional courses offered in those applied science departments which also have a computational science masters and doctoral programs. At present these include, Biology, Physics and Computer Science.

This elective courses naturally draw on many of the course offerings in Mathematics, Scientific Computing, Physics and Computer Science and would provide the student with an opportunity to develop research interests in areas which are not necessarily related to mathematics, or the student could use this opportunity to explore topics in mathematics which are not covered in the curriculum. In addition to the curriculum listed, new elective courses in computational are planned, e.g., in finite

element methods:

1. **Finite element method for solving PDEs, MAT 6xx/7xx.** *Finite element methods (FEM) represent an entire class of methods for solving PDEs, and the aim of this course is to introduce the student to the mathematical theory of FEMs in \mathbb{R}^1 , and provide an introduction to the solution approach in \mathbb{R}^1 and \mathbb{R}^2 .*

- Introduction to variational methods, introduction to the calculus of variation.
- Introduction to the Method of Weighted Residuals (MWR): Bubnov-Galerkin finite elements, and Petrov-Galerkin finite elements.
- Accuracy, and error analysis.
- Developing unstructured grids: Applications in more than one dimension.

Curriculum Objectives and Requirements

This course would naturally draw on many of the course offerings in Mathematics, Scientific Computing, Physics and Computer Science and would provide the student with an opportunity to develop research interests in areas which are not necessarily related to mathematics, or the student could use this opportunity to explore topics in mathematics which are not covered in the curriculum. This course can be seen to benefit the student, and the Department providing a larger base for many of the graduate classes which are offered in the traditional MS program.

4.3 Tools Classes for the MS degree

These courses are offered collectively through the College of Science and Technology for students participating in the Computational Science doctoral program and are discussed in 5.2. These courses are meant to provide the student with an introduction to the techniques and tools which will be useful in writing a thesis or dissertation and to provide them the requisite skills to work with numerical concepts. These courses are not meant to displace traditional computer science or mathematics courses

While a student is required to enroll in only one tools class, it is recommended that the student participate in as many of these as are appropriate for their course of study and background preparation for further study.

4.4 Research Classes

A Masters Thesis provides the serious student with an option to demonstrate the ability to work with and implement numerical methods in a computational environment, and to work with abstract concepts in applied mathematics. For more exceptional students, and for those who are interested in pursuing a PhD in Computational Science in the Department of Mathematics, the thesis option provides the opportunity to get started on their research for the doctorate. This course requires that the MS student be taking the thesis option. It can be taken twice for up to 6 credits.

The thesis research is conducted with faculty supervision, and requires that Thesis, MAT 689 be taken twice:

- Students must arrange to have a graduate faculty member chair their MS thesis committee, and to arrange to have a total of at least three members of the faculty participate on their committees.
- The thesis due dates are subject to the schedules provided by the Graduate Office.

4.5 Considerations for students applying for the MS degree

The MS course outlined should be accessible to students with an undergraduate background which includes in mathematics: multivariable calculus, linear algebra, and a course in the solution of ordinary differential equations; and, at least one course in using the computer as a technological tool for solving computational problems (e.g., a programming course in C or C++, or using Maple or MATLAB for problem solving). Ideally the student should also have had a course in partial differential equations, and a first course in numerical methods would be significant, as would a background in object oriented programming, such as C++, or Java, although this is not required for entering MS students. The student would also require having a BS degree in mathematics, physics, or an applied science such as engineering.

While the tools courses are important in providing the student with a background in utilizing a range of tools suitable for the computational sciences, these courses cannot, in the time provided, substitute for experience in using computers and computer software for problem solving, thus an undergraduate experience which is seriously deficient in these might impair a student's ability to perform well in the proposed degree.

Because the program is designed to be challenging, student preparation for graduate study, as evidenced through scholarly attainment at the undergraduate level, must be examined thoroughly before recommending that a student be admitted to the program. Thus the program is more selective than the traditional MS program in admitting students. Students whose backgrounds are found lacking may be admitted to participate, but will be required to take additional courses, and so will require additional semesters to complete the program.

5 PhD courses

Doctoral students are required to complete at least 18 hours past the MS requirement. For the computational mathematics doctoral student, there are three 3-credit hour core courses, and three three 3-credit hour tools classes which are required in addition to two 1-credit hour seminar courses run through the College of Science and Technology.

5.1 Syllabus for the Core PhD Curriculum

There are only three required core course at the doctoral level. Of these advanced courses, the first continues to develop the underlying theory in applied analysis required to work with and develop

advanced numerical methods while the second develops tools and theory using in scientific computing, particularly as it applies to solving large discrete systems of equations. The third course is in the area of mathematical signal analysis.

1. **CS 741, Functional Analysis for Computational Science.** *Functional analysis with applications in applied and computational mathematics*

- Banach spaces and Hilbert spaces.
- Lebesgue integration and the L^p spaces.
- Foundations of linear operator theory.
- Compact sets in Banach spaces.
- The adjoint operator.
- Linear compact operators.
- The spectral theorem.

<i>Curriculum Objectives and Requirements</i>	This course is an extension of the foundational analytical courses, concepts which are MAT 541/542 Advanced Calculus, I and II, MAT 640 Complex Analysis I, and MAT 641 Complex Analysis I offered to students pursuing a masters in computational mathematics. This course provides a specialized introduction to functional analysis, selecting topics which are of interest to students pursuing advanced studies in computational and numerical mathematics.
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2. **CS 742, Numerical Analysis for Computational Sciences** *Advanced methods in numerical analysis for solving discretized systems*

Error Analysis

- Interpolation
- Integration
- Solving Nonlinear Equations
- Ordinary Differential Equations
- Iterative Method for Large Scaled Problems

<i>Curriculum Objectives and Requirements</i>	This course provides a rigorous follow-up to concepts introduced at the masters level MAT 560 Numerical Analysis, in MAT 605 Differential Equations, MAT 606 Partial Differential Equations, and MAT 610 Numerical Linear Algebra, stressing theory and algorithmics.
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3. **CS 773, Mathematical Signal Analysis** *Techniques for working with standard transform methods, and methods for approximating functions of importance in time series analysis.*

- Fundamentals of analysis in a Hilbert space.
- Heisenberg Uncertainty Principle, and Shanon's sampling theorem.

- Fourier analysis. Plancherel's formula. The FFT.
- Continuous wavelet transforms and frames. The discrete wavelet transform.
- Multiresolution analysis. The scaling function and constructions in the Fourier domain.

<i>Curriculum Objectives and Requirements</i>	The fundamental properties of transform methods, notably centered on the Fourier and Wavelet transform, are developed in this course. The course integrates the analytical ideas developed throughout the masters and doctoral curriculum, developing a sophisticated introduction to the concepts and tools required to work mathematically with time series analysis.
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5.2 Tools Classes for the PhD degree

These courses are offered collectively through the College of Science and Technology for students participating in the PhD Programs in Computational Science. These courses are meant to provide the student with an introduction to the techniques and tools which will be useful in writing a dissertation and to provide them the requisite skills to work with numerical concepts. These courses are not meant to displace traditional computer science or mathematics courses. The tools classes include:

- COS 701, *Presentation and Visualization Tools*: Working with presentation tools, including using LaTeX.
- COS 702, *Data Analysis Tools*: Working with numerical data using basic tools and computational methods which are used to manipulate and analyze large digital data sets. It provides an elementary introduction to using fundamental tools such as signal processing methods for pre- and post-processing data, as well as an elementary introduction to fundamental ideas in numerical and functional approximation to regularize and simplify data.
- COS 703, *Data Storage and Retrieval Tools* Working with file structures in which digital data is stored, and examining the structure of digital data representation including, vector data, and non-numerical data.

Since students participating in the new computational science programs can enter at the MS or PhD levels, these tools classes are constructed so as to be accessible to entry level MS students, and presupposes only a good working knowledge of calculus, the ability to work with basic ideas in algebra and linear algebra, and the ability to work comfortably with computers. There are no computer science programming requirements, however a basic knowledge of computers and experience with using computers and software applications programs is essential.

PhD students are required to enroll in all three tools class.

5.3 Considerations for students applying for the PhD degree

The PhD course outlined should be accessible to students with an strong undergraduate degree in pure or applied mathematics. The graduate background should include those topics discussed in

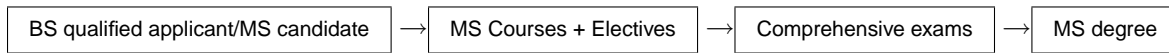
Sec 5.3. Students participating in the doctoral program will spend most of their time interacting with their faculty advisor on research, and so a substantial consideration for any student contemplating the Computational Mathematics doctorate is the research interest and activity of the faculty.

While computational science is often regarded as quite applied, it is, at the level of research mathematics, also quite theoretical, often involving fundamental aspects of pure mathematics more than working with the computer. Nevertheless, to fully appreciate that new methods must be computationally efficient to be useful (or else lead to methods which are), it is important for the student in this field to be able to work with and understand fundamental issues associated with computational algorithmics. The capacity to work with a modern object oriented language such as C++ or Java is of substantial advantage, and so it is strongly encouraged, as is the ability to proficiently utilize symbolic tools such as using Maple or MATLAB for problem solving. The particular course of study requires that the doctoral student have had a course in partial differential equations, and a first course in numerical methods. The student would also require having a BS or MS degree in mathematics, physics, or an applied science such as engineering.

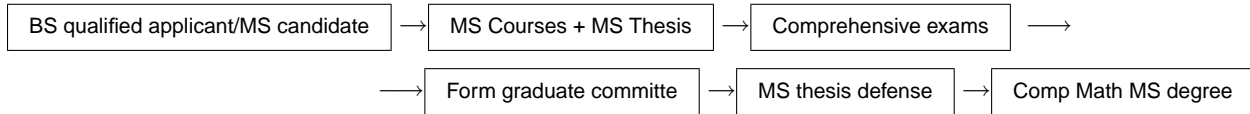
The tools courses are important in providing the student with exposure to interdisciplinary issues associated with computational science, and are also designed to provide some exposure, and hence an ability to work with computational system. These courses are not a substitute for gaining needed proficiency in these areas of computational science.

Because the program is designed to be challenging, student preparation for graduate study, as evidenced through scholarly attainment at the undergraduate level, must be examined thoroughly before recommending that a student be admitted to the program. Thus the program is selective in admitting students. Students whose backgrounds are found lacking may be admitted to participate, but will be required to take additional courses, and so will require additional semesters to complete the program.

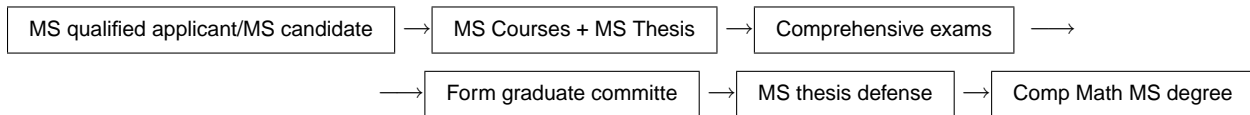
Path 1a: Non-thesis MS option



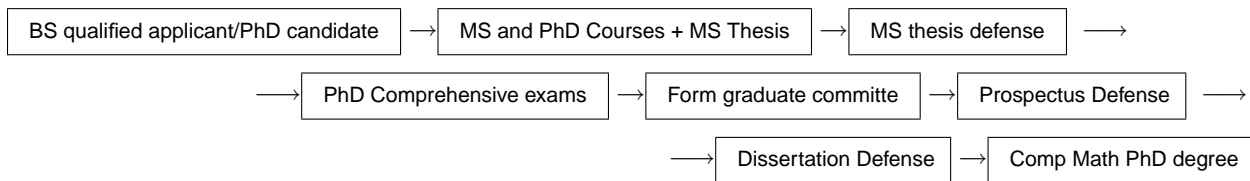
Path 1b: Thesis MS option



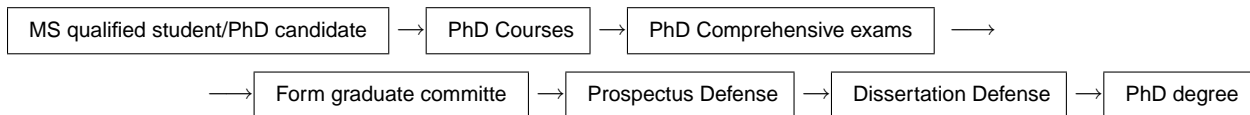
Path 1c: Thesis MS option. Note incoming student's degree cannot be in computational mathematics



Path 2a: Doctoral Program



Path 2b: Doctoral Program



Path 2c: Doctoral Program

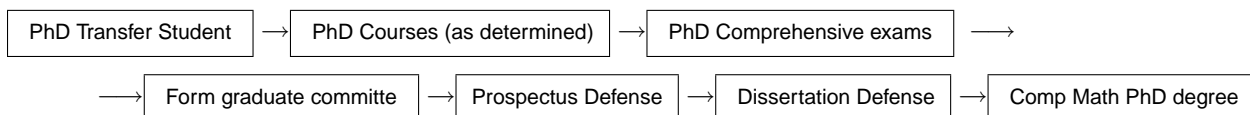


Figure 1: *Progress through the Mathematics MS and Computational Mathematics MS and PhD Program:*

Students may enter with a BS, MS or PhD into the programs of study offered by the Department of Mathematics. Students already holding a doctoral degree in a field other than computational mathematics are considered to be PhD transfer students and will have the curriculum coursework requirements set by their graduate committee.

Students following Path 2, (i.e., those intent on pursuing the PhD degree in Computational Science with emphasis in Computational Mathematics), may terminate at the MS level instead, provided that they have satisfied the requirements for the MS or Computational Mathematics MS degree and have the agreement of their graduate committee.

Students pursuing an MS by research or PhD degree are encouraged to form their graduate committees as early as possible and the timelines shown for formation of graduate research committees is the latest allowable time.

For students following Path 2a whose MS and PhD research topics are the same, may combine the MS Thesis defense with the PhD Prospectus Defense.