PHY 111L Activity 1

Data Analysis

Name: _____ ID #:_____

Lab Section: _____ Date: _____

Lab Partners: _____ TA initials: _____

Objectives

1. Introduce units, conversions, error types, accuracy, and precision
2. Understand the process of measurement and its uncertainties
3. Introduce some of the rules and calculations used when analyzing measurements
4. Experimentally determine π and compare to the theoretical value (lab report)
5. Analyze the validity of experimental results in terms of your measurements

Materials & Resources

1. Pipe segments (various)
2. String
3. Meter stick
4. Digital calipers

Introduction

Units: Definition and examples

At the heart of all scientific inquiry is the notion of a measurable physical quantity that is readily characterized in terms of a specific combination of magnitude and units. The units of the quantity give the magnitude its meaning by relating it to standard or defined values. Some basic physical quantities are time, length, mass and charge. The standard (SI) units for these quantities are seconds (s), meters (m), kilograms (kg), and Coulombs (C), respectively. These most basic quantities can also be combined to represent more complex physical quantities such as:

- velocity with base units $^{length}/_{time}$ and SI units $^{m}/s$ as well as others like mph or fps (see below)
- acceleration with base units $^{length}/_{time^2}$ and SI units $^{m}/s^2$
- force = mass*acceleration with base units $^{mass*length}/_{time^2}$ and SI units N (Newtons)
- impulse (momentum) = force*time (mass*velocity); base units $^{mass*length}/_{time}$ and SI units N*s
- energy = force*length with base units mass*$(^{length}/_{time})^2$ and SI units J (Joules)
- moment of inertia with base units mass*length^2 and SI units kg*m^2

*With others such as electric charge, current, potential and fields, and magnetic flux and fields to be covered in physics 112 next semester.

Units: Conversions

Unit conversions often play an essential role in representing physical quantities; the trick to any unit conversion is to multiply the original quantity by an appropriate representation of the number 1, such that one unit is replaced by an equivalent number of another unit in the next representation of the quantity in question. For example, suppose you are traveling down the highway at a speed of 60 mph (miles per hour) but you want to know how many fps (feet per second) this is. Since there are 3600 seconds in 1 hour and 5280 feet in 1 mile,

$$\frac{1\,hr}{3600\,s} = 1_t \text{ , and } \frac{5280\ ft}{1\,mi} = 1_d \text{. So,}$$

$$60\,mph = 60\,mph \times 1_t \times 1_d = (60\,mph) \times \frac{1\,hr}{3600\,s} \times \frac{5280\ ft}{1\,mi} = 88\ fps\ .$$

Note that the physical quantity of your speed has not been changed by this conversion, but is simply being represented by a different combination of magnitude and units.

Measurement: Errors

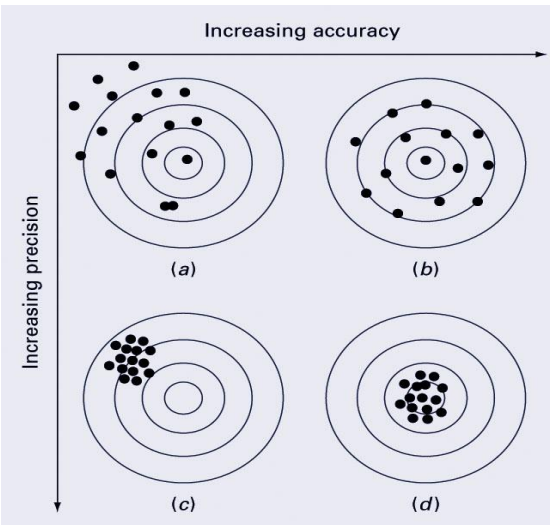
Now, things get a bit more complicated when considering the act of measuring a physical quantity because of the fact that all measurements involve some error and uncertainty. There are several types of error: *personal*, *random*, *assumption*, and *systematic* errors.

<u>*Personal*</u> errors generally involve a subjective bias that is usually based on expected or accepted values for the quantity being measured. This type of error can be largely reduced or completely avoided by being very careful, patient and objective when making measurements. In this lab course, it is expected that you will be careful enough to minimize personal error in your measurements.
It is therefore generally unacceptable to include this type of error in the analysis of your results.

<u>*Random*</u> errors result from unpredictable or uncontrollable experimental conditions such as temperature fluctuations or mechanical vibrations that cause variations from one measurement to the next. This type of error can usually be effectively reduced by taking sufficiently large data sets, or by adding constraints to the experiment designed to control things like temperature and vibration. In this lab course, it is expected that any random errors that may exist will be small enough to neglect when analyzing your results.
It is therefore generally unacceptable to include this type of error in the analysis of your results.

<u>*Assumption*</u> errors are those that result when a simplifying condition like “initial velocity equals zero” is not met in the experiment, or when an assumption used to theoretically characterize a physical system breaks down. These include things like friction from the air, pulleys or wheels of a cart, the mass of strings used, or other physical quantities that are neglected in the theoretical consideration of a physical situation but have an effect on the experimental results nonetheless. For example, when measuring the gravitational constant g, it is often the case that air friction on the object involved is neglected when considering the theory needed to perform the experiment. Although it may be difficult, perhaps even impossible, to know how much this affects the results, it should be clear that the presence of air friction will reduce the value obtained for g, rather than increase it. This type of theoretical shortcoming, combined with the uncertainties associated with the actual measurements, or the amount of dispersion or spread in a set of similar measurements, will both be essential when drawing physical conclusions about your experiments in this lab course.

Systematic errors are those that consistently result from shortcomings in the measurement equipment itself (poorly calibrated, designed, or constructed), such as a force sensor that does not read zero when no force is applied to it, a caliper that has a non-zero reading when it is completely closed, or a stop watch that runs slow because its battery voltage is low. It may or may not be possible to identify these errors in your experiment or to accurately account for them, but they should be included in your analysis if they are present in the experiment and you understand the effect that they have on your results. In general, such errors can be characterized in terms of the accuracy and precision of the measurement device, which refer to how closely the device reads to the actual physical value and how consistently the device reads from one measurement to the next, respectively. This is illustrated in the schematic below:



A good measurement device (*d*) will be both accurate and precise; it will consistently give a reading that is close to the actual value. One (*b*) that is accurate but not very precise will give readings that are spread out from one another but give something close to the true physical value when many readings are averaged together. One (*c*) that is precise but not very accurate will give readings that don't vary much from one reading to the next, but also are not very close to the true physical value, while (a) is neither accurate nor precise. These notions of accuracy and precision are related to the average and standard deviation of a group of similar measurements. This is different from the uncertainty or resolution associated with a single measurement; measurement uncertainty is related to the scale or least count of a given instrument. The Greek symbol Δ ("delta") is used to denote the standard deviation of a group of similar measurements, as well as the uncertainty or resolution of the instrument used for a single measurement.

Analyzing Single Measurements

In this part of the lab, we will make various measurements of a single pipe segment, represent the uncertainty of those measurements using significant figures, investigate the effects of algebraic operations on significant figures, determine the uncertainty in values calculated from those measurements, calculate an experimental value, π_e, for π, use a percent error calculation to compare this to the accepted value π_a, and then examine the validity of our measurements.

1) Locate a pipe segment to be measured, a meter stick, and a set of digital calipers.

2) Using the meter stick, carefully measure the length of the pipe segment and record the result in the following form below: length (L) ± uncertainty in length (ΔL). Be especially careful with your units and significant figures when writing your results.

   - Although it is not necessary that L and ΔL be represented with the same units, doing this is often convenient when calculating uncertainty.
   - Inspection of the meter stick reveals that the least count, or smallest scale division, is 1 mm. The least count of a measuring device is often used to represent the level of certainty about measurements made with that device.
   - Significant figures are used to illustrate the level of physical accuracy of a particular measurement. Since the meter sticks in this lab have a least count of 1 mm, this represents the uncertainty of measurements made with them.

Measured length ($L \pm \Delta L$): _____ ( ) ± _____ ( )

3) Using the digital calipers, carefully measure the inner and outer diameters of the pipe segment and record in a similar form below. Check with your TA if you need help.

- Be sure the calipers are set to mm instead of inches, and calibrate it before each use by closing it all of the way and pressing the "zero" button.
- Use the larger side of the calipers to measure the outer diameter and the smaller side to measure the inner diameter.
- Since the least count of the digital calipers is 0.01 mm, this is the uncertainty associated with this device for both measurements.

Outer Diameter ($D_o \pm \Delta D_o$): _____ ( ) ± _____ ( )

Inner Diameter ($D_i \pm \Delta D_i$): _____ ( ) ± _____ ( )

4) Now that we have the outer and inner diameters of the pipe, we can calculate the thickness of the pipe wall. Since the measurement of the outer diameter includes the thickness of the pipe twice, the thickness of the pipe is given by the following formula:

$$T_p = (D_o - D_i) \div 2$$

- When adding or subtracting measured values with significant figures, the rule is to round the result to as many decimal places as there are in the measured value with the fewest decimal places (not significant figures). For example, 12.34 + 0.8554 = 13.1954 units mathematically, but since this result should only be considered as precise as the 12.34 measurement it should be represented with two decimal places as 13.20 units, where the 5 in the third decimal place rounds up the last significant figure to 0, which is significant.
- When adding or subtracting measured values with associated uncertainties, the uncertainties of the individual measurements should be *added* together to provide the degree of the uncertainty associated with the total calculated value.
- Since the inner and outer diameters are measured with the same device (digital calipers with uncertainty 0.01 mm), the uncertainty of the pipe's thickness is given by 0.02 mm.
- When making calculations, always state the formula or principle used, substitute the values into that formula with the correct units and significant figures, present the purely mathematical result with units, round off the calculated value to the correct number of significant figures and convert the result to the units that are relevant for your situation.
- Calculate the thickness of the pipe and its uncertainty; record your result in the space below with proper units and rounded to the correct number of significant figures.

Pipe Thickness ($T_p \pm \Delta T_p$): _____ ( ) ± _____ ( )

Note: *the lab report for this activity should be based <u>only</u> on sections 5 – 10, <u>not</u> on sections 1 – 4.*

5) Using the string and meter stick, measure the circumference of the pipe by wrapping it around the pipe (as straight as possible and without excess slack), mark the string with a pen or pencil where the string crosses (the thinner the mark the better as long as it is clearly visible), unwrap the string and use the meter stick to measure the circumference and record the result below.

- Notice here that the level of certainty about this measurement depends on the marks made on the string as well as the least count of the meter stick. As long as your marks are not too thick for you to use the 1mm resolution on the meter stick it is still reasonable to use that value for uncertainty.

Circumference ($C \pm \Delta C$): _____ ( ) ± _____ ( )

6) Now that we have measured values for the circumference and diameter of the pipe, we can use the relation $\boldsymbol{\pi} = {}^{C}/_{D}$ to determine an experimental value for pi, π_e. Note: Use D_o from #3 above for D.

- It is important to use all significant figures from the original measured values when substituting into formulas. Keep in mind that even if the last significant figure is measured to be "zero," it is still significant if it is actually read from the measurement device and must be included when considering the significance of values that are calculated from it.
- When multiplying or dividing measured values with significant figures, the rule is to round the result to as many significant figures as there are in the measured value with the least number of significant figures (not decimal places). Be very careful here, the calipers and meter stick give readings that are in different units, 0.01 mm and 1 mm, respectively. The units can be changed by moving the decimal around, but the significant figures do not change since they are based on the measurement device, not what units are used.
- We must look to the decimal place just past the last or smallest significant figure in order to properly round off the result. If this digit is 5 or greater, then the last significant figure gets rounded up. If this digit is 4 or less, then the last figure in the result gets rounded down. For example, 8.0 * 2.934375 = 23.475 mathematically, but since this result should only have two significant figures, it should be represented as 23. Be careful not to round 23.475 to 24 while thinking that the 7 raises the 4 to a 5, which then raises the 3 to a 4.
- It is very important to keep all decimal places throughout all calculations and ONLY round off at the end of the calculation. For example, if you are given a box that measures 2.1 by 4.37 by 0.006, then the volume V = 0.006 * 4.37 * 2.1 = 0.055062 units^3 mathematically and 0.06 units^3 when correctly rounded to one significant figure. But consider that 0.006 * 2.1 = 0.0126 gives 0.01 when rounded to one significant figure, and that 0.01 * 4.37 = 0.0437 gives 0.04 units^3 when rounded to one significant figure, which is quite different than the correct result of 0.06 units^3.
- Show your work for calculating π_e, and record your result, in the space provided below. Present your results rounded to the correct number of significant figures.

π_e = _____

7) Now let's characterize the experimental uncertainty associated with your calculated value for π_e by using the process outlined below. This process will be used often throughout this semester and next!

- Finding the uncertainty of values calculated using multiplication and division is a bit more complicated than when just addition and subtraction are involved, in which case you just add the uncertainties, as in number 4 above.
- First, find the fractional or relative uncertainty, δX, of each measurement by dividing the uncertainty of a particular measurement device by the value of the measurement made by that device. Calculate δC and δD in the space provided below. It is helpful to convert the measured values and the uncertainty in those measurements to the same units before calculating the fractional uncertainty so that the units always cancel.

$\delta C = {}^{\Delta C}/_{C}$ = _____ $\delta D = {}^{\Delta D}/_{D}$ = _____

- Next, square each of these quantities and record in the space provided below.

$(\delta C)^2$ = _____ $(\delta D)^2$ = _____

- Now, adding these two values together and taking the square-root of the total will result in a value for the fractional uncertainty in π.

$\delta\pi_e = \sqrt{(\delta C)^2 + (\delta D)^2}$ = _____ = _____

- But since the fractional uncertainty is always equal to the measurement uncertainty divided by the measured value, or $\delta\pi_e = \Delta\pi_e \div \pi_e$, we can use this relation to find the measurement uncertainty for π, or Δπ, by multiplying the fractional uncertainty, $\delta\pi$, by the value calculated above using $\pi_e = {}^{C}/_{D}$. Do this below, and then record your experimental results in the correct form and with the correct number of significant figures.

$\Delta\pi_e = \delta\pi_e * \pi_e$ = ( ) * ( ) = _____

Experimental value for π: $\pi_e \pm \Delta\pi_e$ = _____ ± _____

- Note: when performing this procedure to determine measurement uncertainties for any calculated results, each measured value and its uncertainty, in the form of its fractional uncertainty δX, must be squared and added to the others under the square-root sign.
- For example, when measuring the volume of a cube with dimensions L, W, and D, the fractional uncertainty in your volume calculation for $V = L*W*D$ will be given by:

$\delta V_{cube} = \sqrt{(\delta L)^2 + (\delta W)^2 + (\delta D)^2}$, where each δ_X represents uncertainty over measured.

- Similarly, when measuring the volume of a cylinder with length L and radius R, the fractional uncertainty in your volume calculation for $V = \pi*L*R^2$ will be given by:

$\delta V_{cylinder} = \sqrt{(\delta L)^2 + \mathbf{2}(\delta R)^2}$; there are **2** "R" terms, since R is squared for a cylinder's volume.

8) Since the accepted value for π is given by $\pi_a \approx 3.14159$, use the following formula to calculate the percent error between your result π_e and the accepted value π_a. Show your work and record your result for percent error in the space provided below using correct significant figures. Ask your TA if you need additional assistance using this formula.

% error = $\left|\frac{measured - accepted}{accepted}\right| * 100\%$ = _____ = _____ %

9) Now that you have calculated the percent error associated with your measurements, which is simply the fraction of the "correct" or "accepted" or "expected" or "theoretical" value that your measurement missed by, let's examine the <u>*validity*</u> of your measurements.

- The first thing to consider when examining the validity of your results is "How much do my results differ from the accepted or theoretical value?" The answer to this question is provided by the simple percent error calculation performed in step 8 above for this case, or in any case where you have a known, accepted, or theoretical value to compare your results to.
- The other thing to consider is the percentage of fractional uncertainty associated with the measurements themselves, which can be found by multiplying the result we calculated in step 7 above by 100%. This value, $\delta\pi_e$ * 100% in this case, answers the question "How much error can I legitimately associate with the measurement equipment that I used to make these measurements?"
- If there is less error in your results than what is accounted for by the uncertainty in your measurement devices, then your results are said to be *valid*, meaning you got as close to the actual or correct result as your measurement equipment would allow you to.
- Another way to say this is: "If your percent error is less than the percentage of fractional uncertainty for a given result, then your result is physically valid." If your percent error is greater than the percentage of fractional uncertainty for a given result, then it can be said that other errors must have been made besides those that are attributable to the limits of the specific measurement equipment used. In which case, you must try to account for these errors in some other way.
- This will often involve an *algebraic analysis* of the equation or physical principle used to determine the results and drawing some <u>*plausible and physically consistent conclusions*</u> about the measurements that were actually made and then used in that equation or principle.

10) For example, suppose that in the above experiment you had a 5% error, but only a 3% fractional uncertainty. Thus, your results are <u>not</u> physically valid since they contain more error than can be legitimately accounted for by the limits of the measurement equipment alone. So, to account for the rest of the error, keep the following guidelines in mind:

- First, was your value higher or lower than the accepted or expected value? This will be very useful, even necessary, when trying to determine a physical basis for what went wrong.
- Second, take a close look at what the formula or equation used tells you about the relationship between your measured values and calculated results. In this case the formula is $\pi = {}^{C}/_{D}$. What this tells you is that if your value for π is too high, for example, then your measurement for C was too high (since it is in the numerator), or your measurement for D was too low (since it is in the denominator), or some combination of both.
- Now that you have a clearer idea of what might have gone wrong, take a close look at what actually happened during the procedure to try to determine what errors might have been involved.
- For example, when measuring C, you wrapped a string around the pipe, marked it with a pen, and then straightened it out to measure the length with a meter stick. If you did not wrap the string precisely straight around the pipe, or if you pulled it tighter when measuring than when it was wrapped, your value for C would have been higher than it should have been, which is consistent with your results and therefore physically plausible.
- Or, if you did not hold the calipers precisely straight when measuring the outer diameter, then your value for D would have also been off. But holding the calipers at any angle when measuring D would have provided you with a result that was larger (rather than smaller) than it should be, which is NOT consistent with you having obtained results for π that are higher than the accepted value. Therefore, this error is not physically plausible in this case.
- This conclusion would be plausible and physically consistent if your calculated result for π_e had been less than the accepted value of 3.14159. In this case, however, the possibility that your string was not straight or that you pulled it too tightly against the meter stick when measuring C would not be a conclusion that is plausible or physically consistent with your actual results.

Questions:

1) Was the value for percent error calculated in number 8 above higher or lower (circle one) than the percentage of fractional uncertainty, $\delta\pi_e$ * 100%?

2) Yes or no (circle one), on the basis of your answer to question number 1, can you consider your results to be physically valid, why?

3) Was the value for π_e calculated in 6 above higher or lower (circle one) than $\pi_a \approx 3.14159$?

4) On the basis of your answer to question number 3, what can you say about the value of your measurement for *C* (in number 5) above? Hint: use the equation for π.

5) Yes or no (circle one), does your answer to question 4 seem physically plausible based on the procedure that you performed in lab? Briefly justify your answer below.

6) On the basis of your answer to question number 3, what can you say about the value of your measurement for *D* (in number 3) above? Hint: use the equation for π.

7) Yes or no (circle one), does your answer to question 6 seem physically plausible based on the procedure that you performed in lab? Briefly justify your answer below.

8) How could you modify the procedure used above to find π to improve the results?

9) Suppose you make 10 measurements with a new device. If all 10 measurements are very close to each other but not very close to the actual or accepted value, would you say the new device is accurate or precise or neither or both (circle one)? What type of error is this?

10) Now suppose the new device made 10 measurements that were very spread out from one another, but all about the same amount from the actual or accepted value, would you say the new device is accurate or precise or neither or both (circle one)? What type of error is this?