PHY 111L Activity 9

Moments of Inertia

Name: _____ ID #:_____

Section: _____ Date: _____

Lab Partners: _____ TA initials: _____

Objectives

1. Introduce moment of inertia for different objects
2. Understand the moment of inertia apparatus and measure its moment of inertia when empty
3. Experimentally determine the moment of inertia of a thick ring
4. Experimentally determine the moment of inertia of a solid disk (lab report)

Materials & Resources

1. Computer with DataStudio and 2 photo gates
2. Moment of inertia apparatus with a ring and disk
3. Balance, meter stick, calipers, hanging mass (about 200 g), pulley, and string

Introduction

Recall that the linear acceleration of an object of mass m is proportional to the net force applied to that object and inversely proportional to its mass. This is often stated as Newton's second law, $\Sigma F = ma$. But for a rotating body (imagine the object is a point mass that is tied to a fixed point by a massless rod of length R) the net torque, τ, that produces an angular acceleration, α, is given by $\tau = RF = R\,ma$, where F is the component of the applied force that is perpendicular to the string. But since the relation between the linear acceleration, a, and the angular acceleration, α, is given by $a = R\alpha$, then

$$\boldsymbol{\tau = \alpha\, mR^2}$$

So, just as m is the "inertia" that relates a net force to linear acceleration, mR^2 is the "moment of inertia" that relates a net torque to angular acceleration. Note that inertia is simply a measure of how much resistance an object has to a change in linear motion (recall Newton's first law); moment of inertia represents a similar resistance for rotational motion. The dimensions are $[M][L^2]$, with SI units $kg{\cdot}m^2$.

If there are many of these point masses on massless rods rotating about a common axis, the moment of inertia of that system is given by $\Sigma(mR^2)$, or the sum of the moments of inertia associated with each mass. If enough masses are tightly packed around the same axis, it is easy to see how that becomes a ring, whose mass, M, is the sum of the masses that comprise it, Σm. The moment of inertia of this ring is given by:

$$\boldsymbol{I_{ring} = MR^2}$$

Similarly, simple calculus shows that the moment of inertia of a solid disk rotating about its center is given by:

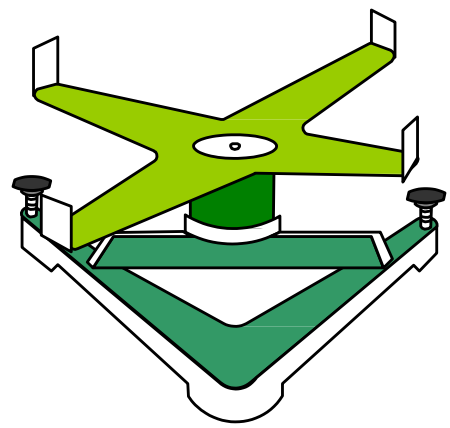
$$\boldsymbol{I_{disk} = \tfrac{1}{2}MR^2}$$

where $M = \Sigma m$ as before. What this means is that a disk of mass M is half as hard to rotate about its axis as a ring of the same mass would be. This is because the farther a mass is from its axis of rotation the farther it must travel to rotate through the same angle as a mass that is closer to the axis, and more of the mass is closer to the axis for a disk than for a ring with the same mass.

In this experiment, we will measure the moment of inertia about the center of mass of both a thick ring and a solid disk, and compare those measurements with the theoretical values given above. We will use gravitational acceleration to impart a torque to the moment of inertia apparatus. But since the apparatus itself is a rotating body, it too has a moment of inertia that must be calculated in order to determine the moments of inertia for the ring and the disk.

1. *Moment of Inertia Apparatus*

Theory of Apparatus:



The illustration to the right shows the rather complicated shape of the apparatus, so rather than directly calculating its moment of inertia, I_A, as above for the ring and disk, let's measure it in the same way that we will use it to measure the ring and disk. Then we can subtract I_A from the measurements for the total moment of inertia when the apparatus is holding the ring or the disk, thereby measuring the moment of inertia of these other objects.

In order to find the moments of inertia of this apparatus, we will need to relate the gravitational acceleration on the hanging mass to the rotational acceleration and moment of inertia of the object in question. A thorough inspection of the figure to the right shows this relation. The linear motion of the hanging mass is governed by Newton's 2nd law, which gives:

$$\Sigma F = mg - T = ma$$

so that the tension in the sting is given by:

$$T = m(g - a)$$

The rotational equation that describes the motion of the shaft on the apparatus is given by:

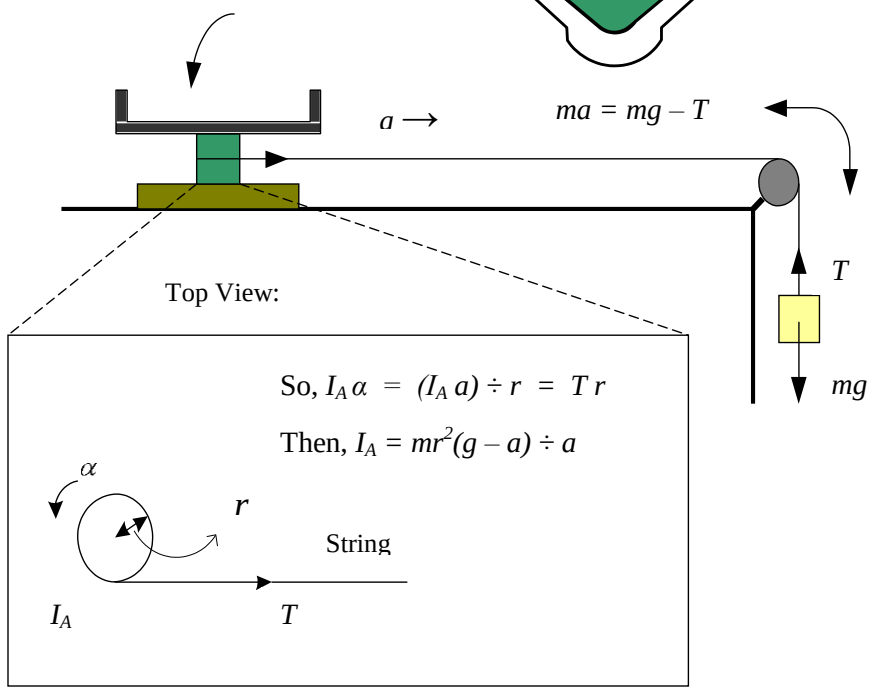
$$I_A \alpha = Tr$$

If we assume that there is no slipping between the string and the rotating shaft of the apparatus, then the relationship between the acceleration of the hanging mass a, which then equals the tangential acceleration of the shaft a_T, and the shaft's rotational acceleration α is given by:

$$a = a_T = \alpha r$$

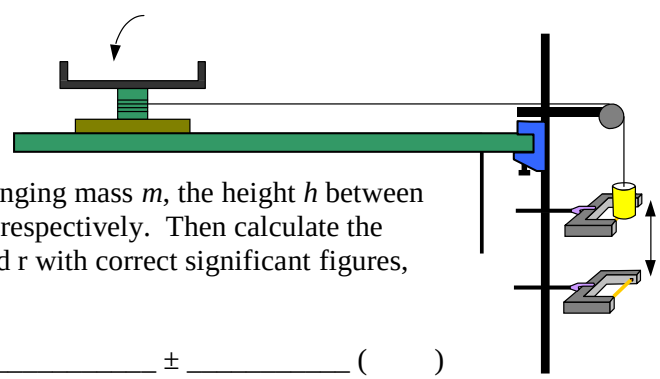
Substituting these expressions for T and α into the equation for I_A and solving for I_A gives:

$$I_A = mr^2(g - a) \div a$$



which relates the moment of inertia of the empty apparatus I_A to the translational acceleration a of the hanging mass due to gravity, which can easily be determined using the vertical travel time t between 2 photo gates, then $a = 2h / t^2$.

Procedure:



1) Set up the empty apparatus with the photo gates, string, pulley, and hanging mass, as shown to the right. Be sure the string is level with the counter and pulley, and that it is long enough for the mass to pass through both gates.
2) Use a balance, meter stick, and calipers, to measure the hanging mass m, the height h between the photo gates, and the diameter d of the apparatus shaft, respectively. Then calculate the radius r of the shaft and record your results for m, h, d, and r with correct significant figures, uncertainty, and units.

   m = _____ ± _____ ( ) d = _____ ± _____ ( )

   h = _____ ± _____ ( ) r = _____ ± _____ ( )
3) Create an experiment in DataStudio and configure the photo gates to measure the travel time of the hanging mass, as in previous experiments. Recall that the initial velocity of the object must be zero, and that the acceleration must be constant, when using the expression $a = 2h / t^2$. Slowly rotate the apparatus to raise and lower the hanging mass when finding the $v_i = 0$ position.

4) Use DataStudio to measure the travel time t of the hanging mass, and then calculate its acceleration a and the moment of inertia of the empty apparatus I_A for each row in the table below. Record your values using correct significant figures and units, where $g = 9.807\ m / s^2$.

Travel time, t	Acceleration, $a = (2h) \div t^2$	Moment of Inertia, $I_A = mr^2(g-a)/a$

5) Enter your values for I_A into a new empty data table, find the average and standard deviation, and record them below. Print the data table showing your entered data and values for average and standard deviation.

<u>Questions</u>: I_A = _____ ± _____ ( )

1) When using this set up to measure moments of inertia, it is assumed that the initial velocity of the hanging mass equals zero, that all friction (from the air, pulley, or wrapped string) can be neglected, and that the string is massless and does not slip around the shaft of the apparatus. Briefly explain how the measured value of the moment of inertia of the apparatus I_A will be affected if each of these assumptions is not really physically accurate.

   In other words, would the measured value for I_A increase or decrease for each condition below, and why?

   Initial velocity $\neq 0$:

   Friction exists:

   String has mass:

   Slipping occurs:

2) Would the measured results for I_A increase or decrease (circle one) if the ends of the apparatus arms were bent *inward* until nearly flat? Justify your answer.

3) Would the measured results for I_A increase or decrease (circle one) if the ends of the apparatus arms were bent *outward* until nearly flat? Justify your answer.

2. *Moment of Inertia of a Thick Ring*

Procedure:

1) Carefully measure the outer diameter of the ring D_R, and then calculate its radius R_R and, using the mass M_R labeled on the ring, its moment of inertia $I_R = M_R R_R^2$. Record these values below using correct significant figures and units.

   D_R = _____ ± _____ ( ) R_R = _____ ± _____ ( ) I_R = _____ ± _____ ( )

2) Place the ring in the *center* of the apparatus; repeat the procedure in part 1 using the table below to find the average and standard deviation of the total moment of inertia I_{TR} of the apparatus and the ring together. Then calculate the moment of inertia of the ring only by using the equation $I_{RE} = I_{TR} - I_A$. Remember to print your table with values.

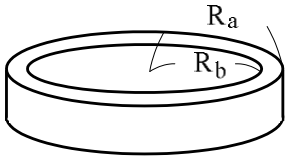
Travel time, t	Acceleration, $a = (2h) \div t^2$	Moment of Inertia, $I_{TR} = mr^2(g-a)/a$

   I_{TR} = _____ ± _____ ( ) $I_{RE} = I_{TR} - I_A$ = _____ ± _____ ( )

3) Note that the equation used in step 1 to calculate the moment of inertia of the ring assumes that the ring has no thickness. Since this ring is too thick for that assumption to be accurate, a better equation to use is given below.

$$I_{R2} = \tfrac{1}{2} M (R_a^2 + R_b^2)$$

   where R_a and R_b are the radii of the outer and inner circles as shown to the right:



4) Since $R_a = R_R$ in this case, measure the inner radius R_b of the ring by either measuring the inner diameter and dividing by 2 or measuring the thickness of the ring directly and subtracting this amount from the measured value for R_R. Record your measured value for R_b below, and then calculate the moment of inertia of the ring using the above formula for a thick ring. Use correct significant figures, uncertainty, and units.

   R_b = _____ ± _____ ( ) $I_{R2} = \tfrac{1}{2} M(R_a^2 + R_b^2)$ = _____ ± _____ ( )

Questions:

1) Which calculated value I_R or I_{R2} (circle one) is closer to the experimental value I_{RE}? Justify your answer below.

2) If the inner radius gets smaller and smaller until it becomes zero, how does this change the equation for a thick ring?

3) What physically happens to the thick ring as the inner radius becomes zero; what shape does it become?

3. *Moment of Inertia of a Solid Disk (Lab Report)*

Procedure:

1) Carefully measure the outer diameter of the disk D_D, and then calculate its radius R_D and, using the mass M_D labeled on the disk, its moment of inertia $I_D = ½M_DR_D^2$. Record these values using correct significant figures and units.

   D_D = _____ ± _____ ( ) R_D = _____ ± _____ ( ) I_D = _____ ± _____ ( )

2) Place the disk in the *center* of the apparatus; repeat the procedure in part 1 using the table below to find the average and standard deviation of the total moment of inertia I_{TD} of the apparatus and the disk together. Then calculate the moment of inertia of the disk only by using the equation $I_{DE} = I_{TD} – I_A$. Remember to print your table with values.

Travel time, t	Acceleration, $a = (2h) \div t^2$	Moment of Inertia, $I_{TD} = mr^2(g-a)/a$

   I_{TD} = _____ ± _____ ( ) $I_{DE} = I_{TD} – I_A$ = _____ ± _____ ( )

3) Calculate the percent difference between I_D and I_{DE}, and then record here:

Questions: *% difference* = _____ %

1) Which value for the moment of inertia of the disk is higher, I_D or I_{DE} (circle one)? Briefly explain why below.

2) When measuring the diameter of the disk above, if the measurement is taken even slightly to one side or the other of the center of the disk, how would this affect your measured value for R_D when compared to the actual radius?

3) How would this affect the moment of inertia calculated for the disk?

4) Do the physical effects of friction or slipping affect the measured results for I_{DE}? Justify your answer below.

5) Are your results for I_D and I_{DE} *physically consistent* with the measurement for R_D being made correctly (through the center of the disk) or incorrectly (circle one)? Briefly justify your answer below.