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THE UNIVERSITY OF SOUTHERN MISSISSIPPI

Mathematics Dept. Student Handbook

Graduate Program

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1 Introduction

This document provides a comprehensive description of

- The masters degree in mathematics, offered through the Department of Mathematics.

- The doctoral degree in Computational Science with an emphasis in mathematics, offered through the College of Science and Technology (CoST) and administered by the Department of Mathematics.

These degrees establish a coordinated applied mathematics program within the Department in which the MS degree can serve as a feeder for the Ph.D. degree and as a strong independent program, in which students with other interests can find a significant range of courses in which to study advanced topics in mathematics. Graduate students have an intensely focused opportunity for developing foundational mathematical skills and for complementing them at the doctoral level with advanced topics in computational mathematical science.

2 The MS Program in Mathematics

At the masters level, the program is designed for students to develop a working background in mathematics. Moreover, students interested in applied mathematics can build the foundation to continue in a doctoral program in applied and computational mathematics. Thus, the masters program provides flexibility to accommodate a wide range of interests as well as the opportunity to specialize in applied mathematics. All MS courses are elective courses without any restrictions. The specific plan of study is developed by the student and the student's graduate committee.

3 The Ph.D. Program in Mathematics

The doctoral degree in Computational Science with an emphasis in mathematics prepares students for research and professional careers in computational mathematics, with an emphasis in applied and numerical mathematics. By its nature, a doctoral degree involves research specialization. To facilitate the specialization in applied and computational mathematics, courses emphasize topics in applied analysis and numerical mathematics.

4 Admissions and Support

4.1 Admission Requirements

To apply for admission, students should have at least earned a bachelor's degree. Students with an appropriate advanced degree can apply directly for the Ph.D.program. Students with an appropriate bachelors degree (BS or BA) must apply for the MS program. After earning an MS degree in mathematics, students interested in further study can then re-apply for the Ph.D. program.

Applicants must meet the requirements specified by the Graduate School posted in the graduate bulletin at http://catalog.usm.edu/index.php: Select the current issue, and look under the section "Admission Requirements and Procedures," "Master's Degree" or "Doctoral Degree." Requirements in addition to the requirements posted there are as follows:

- The undergraduate record should indicate that the applicant satisfies the undergraduate requirements for a major in mathematics at Southern Miss.
- Applicants whose native language is not English must achieve a TOEFL score of at least 580 (TOEFL PBT), or 237 (TOEFL CBT), or 92 (TOEFL iBT), or a 6.5 on the IELTS.

The department chair can recommend conditional admission for an applicant whose credentials strongly meet all requirements (undergraduate record, GRE, and letters of recommendation) except for one.

4.2 Teaching Assistantships

Teaching assistantships are awarded by the chair upon the recommendation of the Graduate Program Committee. Students supported on teaching assistantships are expected to teach or perform other duties for the Department amounting to 20 hours per week during each Fall and Spring semester. All teaching assistants must enroll in the Mathematics Teaching Seminar (MAT 500) during the spring semester of their first year. Full-time support on an assistantship can be provided for up to 3 years for MS students and up to 5 years for Ph.D. students. Students receiving full-time support may not engage in any outside employment. The duties of a first-year teaching assistant include tutoring in the MathZone or in the mathematics tutoring lab. Those first-year students who have previous teaching experience may be assigned to teach one or two sections of Intermediate Algebra (MAT 099) or College Algebra (MAT 101). Second year students usually are assigned as instructor of record for introductory mathematics courses each semester.

5 Student Progress and Degree Requirements

Students pursuing a masters degree are encouraged to write a masters thesis, which entails enrolling in two 3 credit hour research courses. The masters thesis and the associated research work can form the basis for further research for students wishing to continue their studies in the Ph.D. program in Computational Science.

5.1 Progression through the Program

Figure 1 shows the options for progressing through the masters and doctoral programs.

1) Students may enter holding a BS or a graduate degree. Traditional routes, such as bachelors in mathematics to masters in mathematics, bachelors in mathematics to doctorate in computational mathematics, or masters in mathematics to doctorate in computational mathematics, don't require special accommodations. Special cases include:
   - Students holding a bachelors degree in a field other than mathematics, but one which is appropriate for further masters level work in mathematics, can be admitted into the MS degree program.
   - Students already holding a masters degree in a field other than mathematics, but one which is appropriate for further masters level work in mathematics, can be admitted into the MS or the Ph.D. program.
   - Students already holding a doctoral degree in a field other than computational mathematics, but one which is appropriate for further doctoral work in computational mathematics, are considered to be Ph.D. transfer students and will have the curriculum coursework requirements set by their graduate committee.
2) Students pursuing an MS thesis or a Ph.D. degree are encouraged to form their graduate committees as early as possible. All students must have formed their graduate committees by the end of their first year of study. MS committees have at least 3 members, Ph.D. committees have at least 4 members.

5.2 Student graduate committees

Each student's progress is monitored by a faculty advisor. The Department's Graduate Director serves as the advisor for all incoming MS and Ph.D. students. Students are strongly encouraged to begin forming a graduate committee within their first semester at the University. All graduate students are required to have formed their committee by the end of their second semester. The graduate committee typically consists of the chair along with

Path 1a: Bachelors applicants for Non-thesis MS option

MS candidate → MS Courses + Electives → Comprehensive exam → MS degree

Path 1b: Bachelors applicants for Thesis MS option

MS candidate → Start MS Courses + Electives → Form graduate committee
→ Finish MS Courses + Electives + Thesis → MS thesis defense → MS degree

Path 1c: Masters applicants for Thesis MS option. Note incoming student's MS degree cannot be in mathematics

MS candidate → Start MS Courses + Electives → Form graduate committee
→ Finish MS Courses + Electives + Thesis → MS thesis defense → MS degree

Path 2: Bachelors applicant for Doctoral Program taking MS degree

MS candidate → start MS Coursework as determined from transcripts → Form graduate committee
→ finish MS Coursework + Thesis → MS Comprehensive exam → MS thesis defense → MS degree
→ Apply for Ph.D., enter at path 3a

Path 3a: Masters applicant for Doctoral Program

Ph.D. candidate → start Ph.D. Courses → Form graduate committee → continue Ph.D. Courses
→ Ph.D. Comprehensive exams → Prospectus Defense → Dissertation Defense
→ Comp. Math. Ph.D. degree

Path 3b: Masters applicant for Doctoral Program, transfer student

Ph.D. candidate → start Ph.D. Courses → Form graduate committee → continue Ph.D. Courses
→ Ph.D. Comprehensive exams → Prospectus Defense → Dissertation Defense
→ Comp. Math. Ph.D. degree

Figure 1: *Progress through the Mathematics MS and Computational Mathematics Ph.D. Program. Details discussed are meant to exemplify most options, and are not intended to be exclusive.*

at least two other graduate faculty members. The chair of the graduate committee serves as the student's advisor, and is responsible for monitoring the academic progress of the student, and, for MS thesis and Ph.D. students, supervising the student's research. Each Ph.D. student must pass a written Comprehensive Exam based on the Ph.D. Core curriculum before moving on to research status.

Research is the essence of the doctoral degree, which requires the student to master the state of the art in their chosen area of specialization in computational mathematics, *and to extend this state of the art through a research contribution*. Students who enter the doctoral program are encouraged to seek out a faculty research advisor who is willing and able to work with them on a research topic, and who will become the chair of their doctoral committee. The sequence of these activities is illustrated in Fig. 1. The timelines are given in the following subsection.

5.3 Degree Requirements, MS Mathematics

To attain the MS degree, the student must:

1. Fulfill the coursework requirements specified by the Department, including the required minimum number of credit hours to be completed and the requirements for satisfactory performance in all courses.
2. Students choosing the thesis option must:
   (a) Successfully defend the prospectus for the thesis research by the end of their second semester.
   (b) Successfully present and defend the masters thesis.
3. Students choosing the non-thesis option must pass a written comprehensive exam covering courses in their chosen areas of emphasis as shown in Table 3.

The comprehensive exam, prospectus defense, thesis defense, and thesis each must be completed by the appropriate deadline posted on the Graduate School web site, see `http://www.usm.edu/graduate-school/deadlines`. In particular, the MS thesis must be completed before the end of the term in which the student intends to graduate. Students are strongly encouraged to develop ideas, and to approach faculty regarding their research interests early in their studies. Ideally, the student should begin background reading in research late in their first semester.

5.4 Degree Requirements, Ph.D. in Computational Science with an Emphasis in Mathematics

To attain the Ph.D. degree, the student must:

1. Fulfill the coursework requirements specified by the Department, including the minimum number of credit hours to be completed and the requirements for satisfactory performance in all courses.

2. Must successfully defend the prospectus for their dissertation research by the end of the fourth semester.

3. Pass a comprehensive exam covering the core subjects by the end of the sixth semester. The comprehensive exam will cover the core courses associated with the Mathematics Ph.D. degree as shown in Table 2.

4. Complete and successfully defend their Dissertation by the end of the sixth year.

The prospectus defense, dissertation defense, and dissertation each must be completed by the appropriate deadline posted on the Graduate School web site, see http://www.usm.edu/graduate-school/deadlines.

5.5 University MS degree requirements

The Mathematics MS curriculum consists of 30 credit hours, which includes an option for graduate research through the writing of a masters thesis. Students who are pursuing the thesis option may take up to two 3 credit hour courses of graduate research (MAT 698) as part of their 30 credit hours. Courses at or below the 400 level cannot be taken for graduate credit. Any course at or above the 500 level can count toward fulfilling the 30-credit hour requirement.

In addition, MS students are required to have completed 6 hours of Advanced Calculus or equivalent when applying for acceptance into the program. Students who are deficient in this requirement can be accepted provided they complete 6 hours of Advanced Calculus (MAT 541 and 542) within the first two semesters of starting their program of study. These courses will *not* count toward the 30-credit hour requirement.

Of the 30 credit hours counted toward the MS degree, 21 must be from courses numbered 600 and above, and 18 of those must be mathematics courses. For students who choose the thesis option, the two required instances of MAT 698 count toward this requirement.

MS students are strongly encouraged, but not required, to participate in Departmental or College seminars or colloquia, including the COS 740 Computational Science seminar series. These activities are an opportunity to explore research options for students students interested in writing a thesis or continuing in the Ph.D. program. For students following the thesis option, participation in seminars may be required by a student's graduate committee.

The total hours required by the student to complete the MS degree is 2×3 (Advanced Calculus) $+7 \times 3$ (600-level) $+3 \times 3$ (Elective) $= 36$ credit hours. Typically this constitutes a load of 3 graduate courses per semester for 4 semesters.

Foundational MS Courses (24 hrs)	
MAT-641 Functions of a Real Variable I	MAT-642 Functions of a Real Variable II
MAT-636 Functions of a Complex Variable	MAT-601 Differential Geometry
MAT-610 Numerical Linear Algebra	MAT-605 Ordinary Differential Equations
MAT-603 Modern Algebra	MAT-606 Partial Differential Equations

Table 1: MS degree foundational courses. All masters students are required to take 21 hours of 600-level courses, 18 of which must be mathematics courses. These eight courses are generally offered every year or every two years, and are therefore recommended to help fulfill this requirement.

5.6 University Ph.D. degree requirements

Students pursuing a Ph.D. degree are required to take a total of at least 54 credit hours past a masters degree. This amount to a total of 84 credit hours beyond the bachelors degree, of which 30 are in common with the MS curriculum. The doctoral curriculum consists of three 3 credit hour core courses at the 700 level, 12 credit hours of the dissertation course COS 898, along with three 3 credit hour tools classes, and two 1-credit hour seminar classes (COS 740), for a total of $3 \times 3 + 12 + 3 \times 3 + 2 = 32$ credit hours. The remaining 22 hours toward the required 54 past the MS degree can be completed through additional graduate level (500 and above) coursework or additional instances of COS 898.

5.7 CoST Ph.D. degree requirements

The College offers three 3-credit hour courses on research methods, COS 701, COS 702 and COS 703, which are designed to assist the student with technical issues in the area of computing. These "tools classes" are required for all Ph.D. students by the College of Science and Technology. Topics covered include utilizing computational resources, and using standard tools, software libraries, as well as guidance for writing scientific publications. Students entering the doctoral program are expected to have computer skills appropriate for their research interests in computational mathematics as determined by their graduate committee. For doctoral students who need to acquire or augment these skills, it is recommended that elective courses be used to do so.

5.8 Academic probation

The following regulations apply to course grades in the graduate programs.

Core Ph.D. Courses (9 hrs)	Tools Courses (9 hrs)
MAT-771 Functional Analysis MAT-772 Numerical Analysis MAT-773 Signal Analysis	COS-701 Scientific Visualization and Data Mining COS-702 Data Analysis COS-703 Fourier Analysis

Table 2: Ph.D. degree core and tools courses. All doctoral students are required to take these six courses, which constitute a total of 18 credit hours toward the 54 credit hours beyond the MS degree. The core courses form the basis for the written comprehensive exam.

- All MS and Ph.D. students must have passed MAT 541 Advanced Calculus I and MAT 542 Advanced Calculus II, or equivalent, with a grade of B or higher.
- All grades below a grade of B are unacceptable for graduate work, except that one grade of C, not awarded in MAT 541 or 542, may be counted toward the MS degree. No grade of C or lower may be counted toward the Ph.D. degree.

A graduate student receiving either one grade of F or two grades of C during their program of study will be placed on academic probation. Students placed on probation are permitted to retake any graduate course in which they have earned a grade of C or below. If a grade of B or above is earned in the repeated course, the probationary status will be removed.

If the grade requirements cannot be achieved by a student on academic probation, they are terminated from the program.

5.9 Academic honor code

Students have a responsibility to maintain the academic integrity of the University. Therefore, students must conduct themselves in a manner consistent with the University's mission as an institution of higher education and must maintain academic integrity. Violations of academic integrity include, but are not limited to: cheating; plagiarism; unapproved multiple submissions; knowingly furnishing false information to any agent of the University for inclusion in academic records; and falsification, forgery, alteration, destruction, or misuse of official University documents, and misuse or abuse of University facilities. Members of the faculty are responsible for announcing the academic requirements of each course, for the conduct of examinations, and for the security of examination papers.

6 The MS Courses

Students are required at a minimum to complete ten 3-credit courses beyond the Advanced Calculus sequence. Seven of these courses must be 600-level or above, of which two may be research classes for students following a thesis option. Students electing a non-thesis option may not substitute thesis or research classes for other coursework.

The courses offered in the masters program are designed to be accessible to students with an undergraduate background that includes, in mathematics: multivariable calculus, linear algebra, a course in the solution of ordinary differential equations, and, at least one course in using the computer as a technological tool for solving computational problems (e.g., a programming course in C or C++, or using Maple or MATLAB for problem solving). Ideally the student should have had a course in partial differential equations and a first course in numerical methods. The student typically will have a bachelors degree in mathematics, scientific computing, physics, or in an applied science such as engineering, or computational engineering.

6.1 Syllabi for Selected Masters Courses

The masters curriculum is focused on introducing the student to essential ideas in mathematics that cover a large part of modern mathematical thought. As such, it is designed to cover fundamental topics in mathematics, with a focus primarily aimed at providing MS students a solid, classical background in analysis and algebra. This is an essential starting point for work and study in applied and pure mathematics.

Each course is 3 credit hours unless otherwise noted. Brief course description for selected courses are provided below. The descriptions highlight the major topics. At the end of each description, possible connections with other courses in the curriculum are shown and prerequisites are stated. The course descriptions are not meant to be definitive, and are listed in numerical order.

1. **Modern Algebra, MAT 603.** *A selection of topics from the following categories, treated with greater depth and abstraction than in the undergraduate course: groups, rings, modules, algebras, categories, representation theory, and homology.*

   Topics covered include:

   - Monoids and groups.
   - Rings, ideals, and fields. Modules over a principal ideal domain.
   - Category theory.
   - Group representations.

Curriculum Objectives	This course introduces the student to the essential structures of modern algebra. Learning objectives include proficiency in: 1) working in monoids, groups, rings, fields, modules, and categories; 2) recognizing common terminology of algebra, primarily from group theory: and, 3) describing common and unique properties of different groups and rings.
Prerequisite Courses	Modern Algebra II, MAT 424/524.

2. **Differential Equations, MAT 605.** *Extensions of classical theory in the study of ordinary differential equations along with solution techniques.*

   Topics covered include:

   - Solutions for general homogeneous and inhomogeneous linear systems.
   - Real and complex eigenbasis theorems for the constant coefficient linear systems.
   - Stability and linear autonomous systems.
   - Nonlinar systems and the method of successive approximations.
   - The fundamental existence and uniqueness theorem and the maximal interval of existence.
   - Stability and nonlinear autonomous systems.
   - Periodic orbits and limit cycles in planar autonomous systems.
   - Systems depending on a parameter: an introduction of bifurcation.
   - Mathematical models in the form of linear and nonlinear systems of equations.

Curriculum Objectives	The theory of differential equations is quite broad. In this, fundamental concepts from the classical theory of ODEs is developed and broadened to include a study of nonlinear systems of ordinary differential equations with focuses on the qualitative behavior and geometrical theory of the solution set. More specifically, the course is to achieve the following goals: 1) To introduce the fundamental theorem and stability theory for linear systems; 2) to develop the fundamental existence-uniqueness theorem for nonlinear systems; 3) to learn about the qualitative behavior and geometrical properties of solutions of linear and nonlinear systems; 4) to introduce systems of differential equations depending on a parameter; and, to study dynamical systems related to the mathematical model of a physical or biological problem.
Prerequisite Courses	Introduction to Differential Equations I, MAT 285.

3. **Partial Differential Equations, MAT 606.** *The classical theory of partial differential equations along with classical solution techniques.* Topics include:

- Partial differential equations as mathematical models.
- First-order equations and the method of characteristics.
- Classification and canonical form of second-order linear equations in two independent variables.
- The one-dimensional wave equation.
- The method of separation of variables for the wave equation, the heat equation, and elliptic problems.
- Inner product spaces, orthonormal systems, Sturm-Liouville problems and eigenfunction expansions.
- The maximum principle for elliptic equations and the heat equation.
- Green's functions and integral representations.

Curriculum Objectives	To study the partial differential equation problems that arise frequently in engineering and physical applications and to know the foundations of the theory of PDEs for the most fundamental problems in engineering, physics or other sciences. The course is designed to: 1) motivate most mathematical topics by formulating first-order and second-order PDEs from physical principles; 2) classify PDEs into different types and to introduce the theoretical analysis of PDEs for understanding the solution's properties and structure; 3) introduce the integral relations that reduce the number of derivatives required of solutions; and, 4) expose students to the classical techniques of solutions of PDEs including the method of characteristics, eigenfunction expansions, and Green's functions.
Prerequisite Courses	Introduction to Differential Equations I, MAT 285.

4. **Numerical Linear Algebra, MAT 610.** *An introduction to foundational concepts in numerical linear algebra emphasizing theory, but with connections to computational applications.* The course covers:

   - Common problems in linear algebra. Matrix structure and developing methods for efficiently computing with matrix structures.
   - Direct solution methods for linear systems, Gaussian elimination and its variants, including LU decomposition and the Choleksy decomposition. Conditioning, and other numerical difficulties in solving linear systems.
   - Gram-Schmidt procedures, orthogonal reduction, QR factorization, discrete Fourier transforms, Fast Fourier Transforms (FFT), and least squares.
   - Methods for computing eigenvalues such as the Power method and the QR algorithm. Singular value decomposition. Jacobi methods.
   - Iterative solution methods for linear systems including relaxation methods, and the conjugate gradient method. Convergence and rates of convergence of iterative methods.

Curriculum Objectives	This course is intended to cover a broad range of material which includes developing an understanding of the underlying theory motiviating numerical linear algebra. Connections to the need to assess computational and algorithmic efficiency form a significant part of the practical applications examined in classwork.
Prerequisite Courses	MAT 326/426. It is advisable that the student have taken MAT 426/526.

5. **Functions of a Complex Variable I, MAT 636.** *Foundational topics in complex analysis as used in applied mathematics.* Topics covered include:

   - Complex numbers.
   - Analytic functions and elementary functions.
   - Complex integration.
   - Series representations for analytic functions.
   - Residue theory, and the calculus of residues.
   - Conformal mapping.

Curriculum Objectives	The course provides the bridge between the theory of functions of a complex variable and the numerous examples in which this theory is used. The intent is to extend the understanding of analysis using real variables to the complex plane, as well as to introduce some of the unique characteristics of mathemematics done in the complex plain, particularly as applied to foundational problems assocated with calculus.
Prerequisite Courses	Multivariable Calculus, MAT 280.

6. **Functions of a Real Variable I, MAT 641.** *Foundational topics in analysis.*

   - Real numbers, measure sets of real numbers, Borel sets of real numbers.
   - Measure spaces, Legesgue measure.
   - Metric spaces, Lebesgue-Stieltjes Measures, Hausdorff Measures.
   - Measureable functions, sequences of measureable functions, Egoroff's theorem.
   - Integration, Fatou's lemma, properties of the integral, and topic of conergence.
   - Differentiation.

Curriculum Objectives	This course is intended to provide a rigorous introduction to fundamental ideas in real analysis, primarily focusing on the development of measure theory and building from first principals on a framework of elementary analysis accessible to the beginning graduate student.
Prerequisite Courses	Advanced Calculus I, MAT 541, and Advanced Calculus II, MAT 542.

7. **Functions of a Real Variable II, MAT 642.** *Foundational topics in analysis.*
    - Metric Spaces
    - Baire Category Theorem, analytic sets.
    - Banach spaces, the L_p spaces, and Hilbert space.

Curriculum Objectives	This course is continues where Functions of a Real Variable I leaves off, developing foundational topics in analysis, leading to the properties of Banach and Hilbert spaces which are central to modern analysis, and which are part of the framework necessary for further study in analysis, or for developing an understanding of topics in numerical and computational analysis. Again, theory is stressed, working from first principals on a framework of elementary analysis accessible to the beginning graduate student.
Prerequisite Courses	Advanced Calculus I, MAT 541, and Advanced Calculus II, MAT 542.

6.2 Specialty Areas for the MS Degree

Courses offered by the department are grouped into nine specialty areas, which are described in Table 3. With the help of the academic adviser, the student should select a suitable balance in at least three of the nine areas. For the non-thesis student, the master's comprehensive examination will cover the content of two courses from each of the student's three areas of specialization. Courses are selected by the advisory committee in consultation with the student.

6.3 Research class

In a Masters Thesis, the student demonstrates the ability to work with and implement numerical methods, and to work with abstract concepts in applied mathematics. For students interested in pursuing a Ph.D. in Computational Science in the Department of Mathematics,

Topology/Geometry MAT 572 Modern Geometry MAT 575 General Topology MAT 601 Differential Geometry MAT 683 Topics in Topology/Geometry	**Complex Analysis** MAT 536 Theory of Functions of a Complex Variable MAT 636 Functions of a Complex Variable MAT 682 Topics in Analysis
Differential Equations MAT 515 Intro to Differential Equations II MAT 517 Intro to Partial Differential Equations MAT 605 Ordinary Differential Equations MAT 606 Partial Differential Equations MAT 684 Topics in Applied Math MAT 685 Topics in Computational Math	**Linear Algebra** MAT 526 Linear Algebra II MAT 610 Numerical Linear Algebra MAT 681 Topics in Algebra
Computational Mathematics MAT 560 Numerical Analysis I MAT 561 Numerical Analysis II MAT 610 Numerical Linear Algebra MAT 684 Topics in Applied Math MAT 685 Topics in Computational Math	**Combinatorics & Graph Theory** MAT 537 Graph Theory MAT 539 Combinatorics MAT 629 Applied Combinatorics & Graph Theory
Algebra MAT 521 Number Theory MAT 523 Modern Algebra I MAT 524 Modern Algebra II MAT 603 Modern Algebra MAT 681 Topics in Algebra	**Analysis and Probability** MAT 520 Probability & Statistics II MAT 641 Functions of a Real Variable I MAT 642 Functions of a Real Variable II MAT 684 Topics in Applied Math MAT 685 Topics in Computational Math
Optimization MAT 518 Linear Programming MAT 519 Optimization in Math Programming MAT 684 Topics in Applied Math MAT 685 Topics in Computational Math	

Table 3: Grouping of master's courses into specialty areas

the Masters Thesis can be a start on research for the doctorate. MAT 698 can be taken twice for up to 6 credit hours during which the student works on a Masters Thesis.

1. **MAT 698, Thesis** *This course requires that the MS student be taking the thesis option. It can be taken twice for up to 6 credits.*

- Students must arrange for at least three members of the mathematics graduate faculty to serve on their MS thesis committee.

- The thesis due dates are subject to the schedules provided by the Graduate School, see `http://www.usm.edu/graduate-school/deadlines`.

7 Doctoral Courses

Doctoral students in Computational Science with an emphasis in mathematics are required to complete at least 54 hours past the MS degree. There are three required 3-credit hour core courses and three required 3-credit hour tools classes, see Table 2, and two 1-credit hour seminar courses (COS 740).

A substantial amount of time is reserved to pursue an independent course of study as agreed between the student and the graduate committee. The Department offers a range of topics course suitable for advanced graduate students in the area of numerical and applied analysis. These courses can be supplemented with reading courses on topics of interest to the student and the graduate research advisor. Moreover, courses offered at the doctoral level by other departments in the College of Science and Technology can be considered. For example, for students interested in expanding their knowledge of computational topics, there is an extensive curriculum available through the School of Computing, including courses on parallel computing, database structures, and other algorithmic topics that may related to the student's work in computational science.

The Ph.D. core courses should be accessible to students with a strong undergraduate degree in pure or applied mathematics, or with a masters degree in mathematics. The graduate background should include the topics discussed in Sec 6.1. Students participating in the doctoral program will spend most of their time interacting with their faculty advisor on research. Therefore students considering the doctorate in Computational Science with emphasis in mathematics should examine the research interests and activities of the faculty as an indication of possible research topics.

Computational science is often and rightly regarded as quite applied. To fully appreciate that new methods must have computational utility (or else lead to methods which are), it is important for the student to be able to work with and understand fundamental issues associated with computational science. The capacity to work with a modern computer methods, including compiled languages and the ability to proficiently utilize symbolic tools such as using Maple or MATLAB for problem solving, is important. Beyond these aspects, at the level of research mathematics, computational science can become quite theoretical, often involving fundamental aspects of pure mathematics more than working with the computer.

The tools courses (COS 701, 702, 703) expose the student to interdisciplinary issues associated with computational science, and to specific computational systems. These courses are not a substitute for gaining proficiency in specific areas of computational science.

7.1 Syllabi for the Core and Foundational Ph.D. curriculum

The doctoral curriculum overlaps with the masters curriculum and extends it, focusing more on topics in applied and computational mathematics. There are only six 3-credit hour courses at the doctoral level required by the Department of Mathematics: Three required core courses, MAT 771, 772, 773, which are included in the comprehensive exams, and three tools courses, COS 701, 702, 703, which are not. (See Table 2.) Of the core courses, the first continues to develop the underlying theory in applied analysis required to work with and develop advanced numerical methods while the second develops deeper connections computational issues in numerical analysis and scientific computing.

1. **MAT 771, Functional Analysis for Computational Science.** *Functional analysis with applications in applied and computational mathematics*
   - Banach spaces and Hilbert spaces.
   - Lebesgue integration and the L^p spaces.
   - Foundations of linear operator theory.
   - Compact sets in Banach spaces.
   - The adjoint operator.
   - Linear compact operators.
   - The spectral theorem.

Curriculum Objectives	This course is an extension of the foundational analytical courses, concepts which are MAT 541/542 Advanced Calculus, I and II, MAT 640 Complex Analysis I, MAT 641 Functions of a Real Variable I, and MAT 642 Functions of a Real Variable II offered to students pursuing a masters in mathematics. This course provides a specialized introduction to functional analysis, selecting topics which are of importance to students pursuing advanced studies in computational and numerical mathematics.
Prerequisite Courses	Functions of a Real Variable I, MAT 641, and Functions of a Real Variable II, MAT 642.

2. **MAT 772, Numerical Analysis for Computational Sciences** *Advanced methods in numerical analysis for solving discretized systems*
   - Error Analysis, fundamental concepts from mathematics and computers.
   - Approximation and interpolation redone, with attention to theory as well as computational instabilities.
   - Integration techniques.

- Solving nonlinear equations. The theory of fixed point iteration and of relaxation methods, along with some examination of multidimensional issues.
- Numerical solution of ordinary differential equations, with attention to accuracy and stability.
- Iterative methods for large scaled problem, including the conjugate gradient method.

Curriculum Objectives	This course provides a rigorous follow-up to concepts introduced at the undergraduate and graduate levels in MAT 460 Numerical Analysis, in MAT 605 Differential Equations, and MAT 606 Partial Differential Equations, emphasizing the connections between computational mathematics and real and functional analysis. The coursework is designed to put equal emphasis on computational techniques as on theory, and requires the completion of several numerically intensive projects illustrating issues in computational science. Mathematical theory is covered on two levels: 1) foundationally for concepts that are accessible to graduate students, and 2) expository, demonstrating a wealth of techniques and approachs for investigating the behavior of numerical and algorithm issues that arise in computational science.
Prerequisite Courses	MAT 326. It is advisable that the student have taken MAT 426.

3. **MAT 773, Signal Analysis** *Applications of analysis to topics in applied mathematics.*

- The Fourier Transform and distributions.
- The Sobolev spaces and Sobolev inequalities.
- Weak convergence, weak solutions of PDEs.
- Potential thoery
- Elliptic regularity, primarily focusing on the solution of Poisson's equaiton.

Curriculum Objectives	The purpose of this course is to provide connections to advanced topics in applied analysis, particularly as used in the theory of partial differential equations, giving the student exposure to a range of methods and tools for examining issues associated with analytical estimates as well as those associated with convergences.
Prerequisite Courses	Complex Analysis, MAT 636, Functions of a Real Variable I, MAT 641, and Functions of a Real Variable II, MAT 642.

7.2 Tools classes for the Ph.D. degree

The tools courses, offered through the College of Science and Technology for students participating in the Ph.D. Programs in Computational Science, provide the student with an introduction to the techniques, tools and skills that will be useful in computational science. They are not meant to displace traditional computer science or mathematics courses The tools courses are as follows.

- COS 701, *Presentation and Visualization Tools*: Working with presentation and computer based tools.
- COS 702, *Data Analysis Tools*: Working with numerical data using basic tools and computational methods which are used to manipulate and analyze large digital data sets. It provides an elementary introduction to using fundamental tools such as signal processing methods for pre- and post-processing data, as well as an elementary introduction to fundamental ideas in numerical and functional approximation to regularize and simplify data.
- COS 703, *Data Storage and Retrieval Tools* Working with file structures in which digital data is stored, and examining the structure of digital data representation including, vector data, and non-numerical data.

The tools courses are constructed to be accessible to entry level MS students, and presuppose only good working knowledge of calculus, the ability to work with basic ideas in algebra and linear algebra, and the ability to work comfortably with computers and software applications. There are no computer science programming requirements.

Ph.D. students are required to enroll in all three 3-credit hour tools courses.

7.3 Seminar Classes

Students enrolled in the Ph.D. program are required to register and attend two 1-credit hour seminar classes, designated as COS 740.