PHY 111L Activity 10

Rotational Kinetic Energy

Name: ________________________________ ID #:____________________________

Section: _______________________________ Date: ___________________________

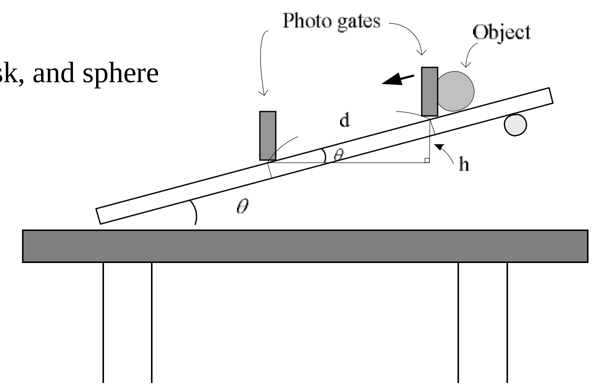
Lab Partners: ____ TA initials: _______

Objectives

1. Introduce rotational kinetic energy in terms of moments of inertia
2. Use the Work-Energy Theorem to determine the final velocity of objects rolling down an inclined plane
3. Experimentally determine the velocity of a ring, disk, and sphere and compare to calculated values (lab report)

Materials & Resources

1. Computer with DataStudio and 2 photo gates
2. Protractor, meter stick, and an inclined plane with a ring, disk, and sphere



Introduction

Recall that the translational kinetic energy (*TKE*) of a body is given by:

$$KE_t = \tfrac{1}{2}mv^2$$

Similarly, the rotational kinetic energy (*RKE*) is expressed as:

$$KE_r = \tfrac{1}{2}I\omega^2$$

where I is the body's moment of inertia, and ω is the angular speed. The total kinetic energy of an object in general consists of both translational and rotational energies. Recall that the work-energy theorem (lab #5) states that the change in potential energy equals the change in kinetic energy for a conservative system, or one in which no external forces are acting. When a body is both translating and rotating, the Work-Energy Theorem becomes:

$$KE_t + KE_r = PE$$

What this means is that, for example, some of an object's gravitational potential energy may end up as rotational kinetic energy as well as translational kinetic energy. Under these conditions, the work-energy theorem gives:

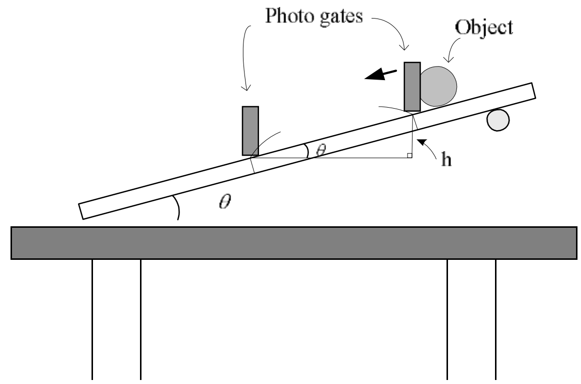
$$\tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2 = Mgh$$

Now, for an object that rolls without slipping, $v = R\omega$, where v is the translational velocity of the object, ω is its angular velocity, and R is its radius. Then, $\omega = {}^{v}/_{R}$. Substituting and solving the energy equation for the expected value for v_e gives:

$$v_e = \sqrt{\frac{2gh}{1 + \frac{I}{MR^2}}}$$

where M is the mass, R is the radius, and I is the moment of inertia of the object, while h is the vertical (in gravity's direction) distance traveled by the object. Note that I is divided by a factor of MR^2 in the formula to leave $1 + f$, where f is a fraction associated with the object's shape. As in previous labs, we will setup a timer using two photo gates and use the acquired data to find the velocity associated with the kinetic energy of various objects. Recall that since gravity provides a constant acceleration over short distances near the surface of the Earth, the average velocity (${}^{\Delta d}/_{\Delta t}$) equals $(v_f + v_i) \div 2$. In order to simplify this further, set $v_i = 0$ by placing the object right next to the first photo gate so that it has no velocity as it starts to pass it. Then $v = 2d \div t$. We will measure this with the photo gates and compare it to v_e.

1. *Velocity of a Rolling Ring*



Procedure:

1) Set up the experiment as shown to the right. Measure the angle θ using the angle indicator attached to the track and the travel distance d using the scale on the track or a meter stick. Calculate the horizontal distance h and its uncertainty Δh. Record your results below with correct significant figures, uncertainty, and units. Show all of your work for the h and Δh calculations in the space provided to the right.

$\theta \pm \Delta\theta$ = ________ ± ______ (   )

$d \pm \Delta d$ = ________ ± ______ (   )

$h \pm \Delta h$ = ________ ± ______ (   )

3) Measure the mass of the ring, disk, and sphere; record with correct significant figures, uncertainty and units below.

$M_r \pm \Delta M_r$ = _______ ± _____ (   ) $M_d \pm \Delta M_d$ = _______ ± _____ (   ) $M_s \pm \Delta M_s$ = _______ ± _____ (   )

4) Use the digital calipers to measure the inner diameter D_i and the outer diameter D_o of the ring, and then calculate the inner radius R_i and the outer radius R_o of the ring, as well as its moment of inertia using the equation for a thick ring $I_r = ½ M_r(R_i^2 + R_o^2)$ and its expected velocity v_{er} at the second gate using the equation derived in the Introduction above. Record your measured results with correct significant figures, uncertainty, and units; and your calculated results with correct significant figures and units below. Show your calculations for I_R and v_{er} below.

$D_i \pm \Delta D_i$ = _______ ± _____ (   ) $D_o \pm \Delta D_o$ = ________ ± _____ (   )

$R_i \pm \Delta R_i$ = _______ ± _____ (   ) $R_o \pm \Delta R_o$ = ________ ± _____ (   )

I_R = ___________________ (   ) v_{er} = _____________________ (   )

5) Create an experiment in DataStudio with 2 photo gates and select a table that displays the time between any gates. Roll the ring through the gates at least 10 times; be sure that the initial velocity of the ring is zero so that the calculated final velocity is accurate, as in previous labs. Also take care to ensure the track does not move so that the angle stays the same for each run. Find the mean and standard deviation of the travel time for the ring t_r using the "Σ" functions, then calculate its velocity at the second photo gate using $v_r = 2d / t_r$. Print the completed table to include in your lab report. Calculate the percent difference between v_r and v_{er} and record your results below.

$t_r \pm \Delta t_r$ = _______ ± _____ (   ) $v_r \pm \Delta v_r$ = _______ ± _____ (   ) % difference for v_r and v_{er}: ________ %

Questions:

1) Is your measured value for v_r approximately equal to the expected value v_{er} for the ring, yes or no (circle one)?
2) Which value for the final velocity of the ring is higher, v_r or v_{er} (circle one)?
3) Briefly justify this experimental result in terms of the measurements made in order to calculate v_r (d and t_r).

4) Briefly justify this result in terms of any of the measurements needed to calculate v_{er} (d, θ, M_r, D_i, and D_o).

2. *Velocity of a Rolling Disk*

1) Use the digital calipers to measure the diameter D_d of the disk, calculate its radius R_d, and record your results below with correct significant figures, uncertainty, and units. Calculate its moment of inertia using $I_d = ½M_dR_d^2$, as well as its expected velocity v_{ed} as in step 4 above. Show your calculations for I_d and v_{ed} in the space below.

   $D_d \pm \Delta D_d$ = __________ ± ________ (   )

   $R_d \pm \Delta R_d$ = __________ ± ________ (   )

   I_d = ___________ (   ) v_{ed} = ___________ (   )

2) Repeat the procedure outlined in step 5 above to find the average and standard deviation of the travel time of the disk, as well as its velocity at the second photo gate using $v_d = 2d / t_d$. Print the completed table to include in your lab report. Calculate the percent difference between v_d and v_{ed} and record your results below. Note: do not confuse the travel distance d with the subscripts that refer to the disk.

   $t_d \pm \Delta t_d$ = ______ ± _____ (   ) $v_d \pm \Delta v_d$ = ______ ± _____ (   ) % difference for v_d and v_{ed}: ________ %

3. *Velocity of a Rolling Sphere (Lab Report)*

1) Use the digital calipers to measure the diameter D_s of the sphere, calculate its radius R_s, and record your results below with correct significant figures, uncertainty, and units. Calculate its moment of inertia using $I_s = {}^2\!/_5\, M_sR_s^2$, as well as its expected velocity v_{es} as above. Show your calculations for I_s and v_{es} in the space below.

   $D_s \pm \Delta D_s$ = __________ ± ________ (   )

   $R_s \pm \Delta R_s$ = __________ ± ________ (   )

   I_s = ___________ (   ) v_{es} = ___________ (   )

2) Repeat the procedure above to find the average and standard deviation of the travel time of the sphere, as well as its velocity at the second photo gate using $v_s = 2d / t_s$. Print the completed table to include in your lab report. Calculate the percent difference between v_s and v_{es} and record your results below.

   $t_s \pm \Delta t_s$ = ______ ± _____ (   ) $v_s \pm \Delta v_s$ = ______ ± _____ (   ) % difference for v_s and v_{es}: ________ %

<u>Questions</u>:

1) For which object did you calculate the largest percent difference for v_x and v_{ex}? Provide a brief physical or experimental justification for this result below.

2) If a sphere, a disk, and a ring with the same mass and radii, all begin traveling down the same slope at the same time, which would arrive at the bottom first, second, and third? Why?

3) Why is the moment of inertia different for each object, and how does this affect each object's rotational energy?

4) Does the final velocity of each object depend on its mass? If so, how? If not, why?

5) How would the results be affected if the objects did have an initial velocity? Justify your answer.