PHY 112L Activity 4

Capacitance and the RC Time Constant

Name: _________________________________  ID #: __________________________
Section: ________________________________  Date: __________________________
Lab Partners: ________________________________  TA initials: ________

Objectives
1. Introduce capacitance and RC time constants for charging and discharging capacitors
2. Experimentally determine the capacitance of different capacitor configurations
3. Experimentally determine the time constant of an RC circuit and compare to calculated values (lab report)

Materials & Resources
1. Computer with Data Studio, Pasco interface, and voltage sensor
2. Capacitance module, multimeter, and power supply

Introduction

Another commonly used electronic component is the capacitor. Capacitors have a different physical property than those of resistors. This property, called capacitance $C$, describes the capacitor's ability to store electrical charge. The SI unit of capacitance is the farad (F). Charge, capacitance and voltage are related by

$$Q = CV$$

This equation says that the amount of charge stored in a capacitor $Q$ at any given time can be calculated by multiplying the capacitance $C$ of the capacitor and the voltage difference $V$ measured across it.

Like resistors, capacitors can be configured in parallel and series to form equivalent capacitances $C_P$ and $C_S$, respectively. If two capacitors are connected in parallel, then

$$C_P = C_1 + C_2.$$  

This is because the voltage on both capacitors is equal; only the charge differs according to the value of each capacitor. On the other hand, if the capacitors are connected in series, the charge on both capacitors is equal, but the voltage differs according to the value of each capacitor. In this case, the equivalent capacitance is given by

$$C_S = \frac{C_1 C_2}{C_1 + C_2}.$$  

A circuit that contains both a resistor and a capacitor is called an RC circuit. Simple RC circuits can be used to illustrate the charging and discharging properties of a capacitor. When you apply a voltage across a capacitor, it will begin to store charge at a rate that depends on both the resistance $R$ and the capacitance $C$ of the circuit. Similarly, when the charged capacitor is configured to discharge, it will do so at the same rate because of this dependence on $R$ and $C$.

The time in which the charging or discharging occurs can be extremely rapid in computers or other common devices, but here $R$ and $C$ are chosen to make charging and discharging easy to observe. The charging and discharging formulas are:

Charging: $Q = Q_m \left(1 - e^{-\frac{t}{RC}}\right)$  

Discharging: $Q = Q_o e^{-\frac{t}{RC}}$

where $Q_m$ is the maximum amount of charge that can be stored within the capacitor, $t$ is the time elapsed since the voltage was turned on (for charging) or off (for discharging). Inside the exponential function, $RC$ characterizes how fast the capacitor stores charge and is referred to as the circuit’s time constant, $\tau = RC$. This is how long it takes for the capacitor to charge to 63% of its maximum value $Q_m$ or to discharge to 37% of the initial amount of charge $Q_o$. It is very useful to be able to control the timing of a circuit’s charging and discharging by choosing different values for $R$ and $C$!
1. Measuring Capacitance and Equivalent Capacitance

Procedure:

Section 1 - Capacitor 1 (C₁):

1) Start DataStudio, setup the voltage sensor on an open input port, and then display a voltage vs. time graph.
2) Connect the voltage sensor leads to the capacitance module as shown in Figure 1 (red to J₃, black to J₁, and jumper between J₁ and J₄). In this configuration, you will be measuring the voltage across C₁.
3) Connect the power adapter to the power plug socket.
4) Reset the capacitance module. This is done by changing S₁ to Off, S₂ to Discharge, and S₃ to R₀, where there is no resistor.

Note: when resetting the capacitor, it must discharge for several seconds in order to remove all of the stored charge.
5) Select R₁ on S₃ and Charging on S₂, and then click “Start” in DataStudio, and then flip S₁ to On.
6) As the capacitor charges, the voltage will eventually approach a maximum value. Stop data collection when it reaches this point and reset the capacitance module as in step 4 above.
7) Select the range on the line that appears linear (a straight line) by highlighting it as shown in Figure 2, then select “Linear Fit” from the “Fit” menu. Record your results for slope m in column 1 of the table to the right, and then print this graph.

This graph is a plot of voltage vs. time. In order to calculate C₁, the charge and voltage are needed. The charge pump for the capacitance module delivers a steady current of 1.0 x 10⁻³ A. Thus, the equations I = ΔQ/Δt and Q = CV can be used to interpret the slope m like this:

\[ m = \frac{ΔV}{Δt} = \frac{ΔV}{ΔQ} \cdot i = \frac{i}{C}. \]

So, capacitance equals current divided by the slope of the measured voltage vs. time plot, or simply \( C = \frac{i}{m} \).

<table>
<thead>
<tr>
<th>Slope from graph (m)</th>
<th>Capacitance in (F) C = I / m (where I = 1x10⁻³ A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td></td>
</tr>
<tr>
<td>C₂</td>
<td></td>
</tr>
<tr>
<td>CEP</td>
<td></td>
</tr>
<tr>
<td>CES</td>
<td></td>
</tr>
</tbody>
</table>

Questions:

1) Briefly describe the voltage vs. time graph of an RC circuit when the charge stored on C is approaching...

   a) its maximum?

   b) zero?

2) Why is the slope of the voltage vs. time graph in Figure 2 a straight line instead of curved like an exponential?
Section 2 – Capacitor 2 ($C_2$):

8) Delete your data for $C_1$ by selecting “Experiment” and then “Delete ALL Data Runs.”

9) Connect the voltage sensor across $C_2$ (red plug to $J_4$, black plug to $J_2$, jumper across $J_1$ and $J_2$).
   In this configuration, you will be measuring the voltage across $C_2$.

10) Repeat steps 4 – 7 above to find the capacitance of $C_2$.

Section 3 – Capacitors in Parallel ($C_{EP}$):

11) Modify the connections to the capacitance module as follows:
   Leave the jumper across $J_1$ and $J_2$ in place, and reconnect a jumper across $J_3$ and $J_4$ as in Part 1 above.
   Also, connect the voltage sensor across $C_1$ as described in Part 1. *Note, since the capacitors are now connected in parallel, the voltage measured here is the voltage across both capacitors in parallel.*

12) Delete your data for $C_2$ and repeat steps 4 – 7 above to find the capacitance of $C_1$ and $C_2$ in parallel, $C_{EP}$.

Section 4 – Capacitors in Series ($C_{ES}$):

13) Modify the connections to the capacitance module as follows:
   Remove the voltage sensor from the circuit and disconnect all jumpers. Connect the red plug of the voltage sensor to $J_4$ and the black plug to $J_1$, then connect a jumper across $J_2$ and $J_3$. *Note, the capacitors are now connected in series, and the voltage is measured across both capacitors in series.*

14) Delete your data for $C_{EP}$ and repeat steps 4 – 7 above to find the capacitance of $C_1$ and $C_2$ in series, $C_{ES}$.

15) Using the equivalence formulas and the values measured for $C_1$ and $C_2$ above, calculate $C_P$ and $C_S$; record below.

\[
C_P: \text{________________________} = \text{__________} \quad C_S: \text{________________________} = \text{__________}
\]

16) Calculate percent differences for both the parallel and series capacitor configurations; record your results below.

\[
\% \text{ difference (parallel)} = \frac{|C_{EP} - C_P|}{\frac{1}{2}(C_{EP} + C_P)} \times 100 = \text{________________________} = \text{__________} \%
\]

\[
\% \text{ difference (series)} = \frac{|C_{ES} - C_S|}{\frac{1}{2}(C_{ES} + C_S)} \times 100 = \text{________________________} = \text{__________} \%
\]

Questions:

1) Why must C discharge completely between different measurements of $C_X$? How can this be verified experimentally?

2) How would your measured values for $C_S$ change if there was an initial charge on $C_1$ or $C_2$? What about for $C_P$?

3) Is your calculated value for $C_P$ higher or lower than your measured value $C_{EP}$? What about your $C_S$ and $C_{ES}$?

4) What quantity represents the uncertainty in the above method for measuring values for $C_X$?
2. **RC Circuit Time Constant (Lab Report)**

**Procedure:** (The connections for the capacitors in series will be reused here; connection changes are not needed.)

1) Delete your data for $C_{ES}$ by selecting “Experiment” and then “Delete ALL Data Runs.”

2) Reset the capacitance module and then begin charging the series combination of $C_1$ and $C_2$ again by repeating steps 4 and 5 from Part 1 above.

3) Allow the capacitor to charge until the voltage vs. time graph flattens out at its maximum value $V_{\text{max}}$. This may take several seconds; be sure the voltage is flat before proceeding.

4) Turn $S_2$ to Discharge and allow the capacitor to discharge until the voltage has fallen to zero again. Then stop data collection and reset the capacitance module.

5) Use the Smart Tool in DataStudio to find the corner at the peak of the discharge curve as shown in Figure 3. This will give the precise time that the discharge began $T_o$ as well as the maximum voltage for this capacitor $V_{\text{max}}$ in the form $(T_o, V_{\text{max}})$. Record your measured values for $T_o$ and $V_{\text{max}}$ in the space below.

\[ T_o = \quad \text{___________} \quad (\quad ) \quad V_{\text{max}} = \quad \text{___________} \quad (\quad ) \]

6) When the voltage across a discharging capacitor has fallen to 37% of its maximum value, one time constant is said to have passed. Calculate 37% of the maximum voltage $V_{37}$ and locate this point on the curve as in Figure 4. Record your results below.

\[ T_{37} = \quad \text{___________} \quad (\quad ) \quad V_{37} = \quad \text{___________} \quad (\quad ) \]

7) Use the Smart Tool to locate $V_{37}$ on the discharge curve as accurately as possible, as shown in Figure 4, and then record the time at this point $T_{37}$ in the space above.

8) Calculate and record the time constant $\tau_E$ using $T_o$ and $T_{37}$.

\[ \tau_E = T_{37} - T_o = \quad \text{___________} \quad = \quad \text{___________} \quad (\quad ) \]

9) Turn the multimeter to resistance mode, and then measure the resistance of $R_1$ as shown in Figure 5 record below.

\[ R_1 = \quad \text{___________} \quad (\quad ) \quad \tau_C = \quad \text{___________} \quad (\quad ) \]

10) Calculate the time constant $\tau_C = R_1C_{ES}$ using $R_1$ measured here and $C_{ES}$ measured in Section 4 above and record your result above. Calculate the percent difference below.

\[ \% \text{ difference} = \frac{|\tau_E - \tau_C|}{\frac{1}{2}(\tau_E + \tau_C)} \times 100 = \quad \text{________________________} = \quad \% \]

**Questions:**

1) Which value for time constant is higher, $\tau_E$ or $\tau_C$? Briefly explain why below.

2) How would your measured values for $\tau$ above change if $R_2$, which has a higher resistance than $R_1$, is used instead?